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Fractal model to characterize spontaneous imbibition in porous media

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Abstract: Spontaneous imbibition is an essential phenomenon that exists in a many enhanced oil recovery processes. The imbibition of wetting liquid into porous media is an important driving mechanism for oil recovery. In this study, an analytical model was developed for the characterization of spontaneous countercurrent imbibition process of wetting fluid into gas-saturated rock over the entire time of imbibition process by utilizing the fractal characters of porous sediments. The weight of the imbibed liquid is a function of the pore fractal dimensions and contact area, porosity, tortuous capillaries, hydraulic diameter of pores, porosity, fluid properties, surface tension, as well as the fluid-solid interactions. The validity of the model was tested using published experimental data horizontal spontaneous imbibitions in g–water–rock systems. The model predictions are consistent with the published experimental data.

Introduction

Spontaneous imbibition is phenomenon that occurs naturally in many fields such as petroleum engineering, engineering geology, groundwater engineering, civil engineering and soil physics (Cai *et al*., 2012). It is the principal mechanism of fluid flow in several oil reservoirs, mostly those developed by water injection **(**Li and Zhao**,** 2012**)** and/or waterflooding (Li and Horne, 2001). The imbibition of wetting fluid spontaneously into porous rock is a key driving mechanism for enhancing the recovery of oil from many reservoirs particularly those with low permeability (Cai *et al*., 2012; Fernø, 2012). Spontaneous imbibition study is crucial in the evaluation of the wettability of the solid-liquid systems (Cai, *et al.*, 2010a) as well as in the prediction of the production performances in oil/gas reservoirs. The imbibition rate is normally related to fluid properties and their interactions together with the microstructures of porous sediments. These interactions includes absolute and relative permeability (Li and Horne, 2001), viscosity (Standnes, 2009), initial water saturation (Schembre *et al*., 1998), shapes and boundary conditions (Zhang *et al*., 1996), wettability and interfacial tension (Cai, et al., 2010a). Several researchers have studied spontaneous imbibition in porous media in the past years (Lefebvre Du Prey 1978; Al-Lawati and Saleh 1996; Zhang *et al*., 1996; Babadagli, 2001; Høgnesen *et al*., 2004; Tavassoli *et al*., 2005). In this work, a review of many models reported in the Literature was undertaken and a fractal model to characterize spontaneous imbibition and study fluid flow mechanisms was proposed by taken into account the property of the rock represented by fractal dimension. Washburn (1921) proposed a simple cylindrical model by theoretically analysis of spontaneous water imbibition which was based on mass balance and continuum equations. In his model, the porous sediment is represented by a collection of parallel tubes, with the same radius. The pore sizes in natural porous rocks vary at different points and they extend several orders of magnitude. Washburn proposed that the depth of penetration (h) of the wetting fluid into a porous sediment is proportional to the square-root of time (t):

$$h = St^{1/2} \qquad 1.1$$

where S is the sorption constant. When the flow is horizontal in constant radius pores, the constant (S) is given as follows:

$$S = \sqrt{\frac{r\sigma cos\theta}{2\mu_w}} \qquad 1.2$$

where r is radius of the pore, σ is interfacial tension, θ is contact angle between the wetting and non-wetting fluids, and μ_w is the viscosity of the wetting fluid. The model failed to properly quantify spontaneous imbibition process, due to the complex structure of natural porous sediments. Benavente *et al.,* (2002) modified the Lucas-Washburn (LW) equation (Lucas, 1918; Washburn, 1921) by the introduction of different corrections that relates to the microstructures (tortuosity and pore shape) of the porous media. They introduced tortuosity in their model and assigned 3 to it as an empirical constant. Cai *et al.,* (2010a) pointed out that, the assumption was unreliable since tortuosity normally varies with effective porosity and microstructures of porous sediments. Additionally, their experimental data did not strongly agree with the predictions of their model. Handy (1960) also proposed a model to characterize spontaneous water imbibition for gas-liquid-rock systems by assuming piston-like displacement of the imbibition process. Gravity was ignored in the derivation of his model and the mobility of the gas phase was assumed infinite. His equation states that the square of the volume of imbibed water is proportional to the imbibition time:

$$N_{wt}^2 = \frac{(2P_c K_w \emptyset A^2 S_w)}{\mu_w} t \quad 1.3$$

where N_{wt} is the total volume of water imbibed into the core, Ø is the porosity, μ_w is the viscosity of water, A is the cross sectional area of the core, K_w is the effective permeability of water phase at water saturation of S_w, S_w is the water saturation behind the imbibition front, t is imbibition time and P_c is the effective capillary pressure at S_w. Li and Horne, (2001) pointed out that from eq. (3), when imbibition time approaches infinity, the amount of water imbibed into the porous media will also be infinite, and this is practically impossible for all systems, not only for vertically mounted systems. Furthermore, effective water permeability and capillary pressure cannot be calculated separately from a spontaneous water imbibition test; that is permeability and capillary pressure must be determined separately. From their experimental results, they stated that depending on the ratio of gravity to the capillary pressure gradient, gravitational forces should not be ignored in many cases. They modified eq. (1.3) based on the hypothesis that the spontaneous imbibition of water rising vertically in a core sample follows Darcy's law by taking into account the effect of gravity and thus the infinite value from eq. (1.3) would vanish. Additionally, permeability and capillary pressure would be determined independently from test data, through the use of their proposed semi-empirical correlations for water imbibition into porous sediments. Laughlin and Davies (1961) proposed a model through the measurement of capillary rise of oil in fibrous textile wicking:

$$R = \Omega t^m \quad 1.4$$

where R is the recovery by spontaneous imbibition, Ω is a constant related to rock/fluid properties which includes viscosity, porosity, capillary pressure, permeability and relative permeability, m is the exponent and t is the production time. They discovered that the value of m varied from 0.41 to 0.50. Zimmerman and Bodvarsson (1991) reported a model similar to the Washburn equation:

$$V(t) = \omega t^{1/2} \quad 1.5$$

where $v(t)$ is the cumulative water flux. The constant ω is computed using the equation below:

$$\omega = \left(\frac{2k\emptyset(S_{wm} - S_{wm})}{\alpha_e \mu_w}\left[1 + \frac{(S_{wm} - S_{wm})}{2\lambda(S_{wm} - S_{nwr})}\right]\right)^{1/2} \quad 1.6$$

where S_{wm} is the saturation of the wetting phase at zero potential or the water saturation, S_{nwr} is the residual saturation of the non-wetting phase, λ is the pore size distribution index and α_e is capillary pressure scaling parameter.

Several researchers (Katz and Thompson, 1985; Krohn, 1988; Toledo *et al*., 1993; Li, 2004a) have also studied the fractal behaviour of different porous media for more than two decades and it was found that most natural porous sediments such as reservoir rock and other porous materials are fractals (Li, 2004b). There is therefore, a general acceptance that porous media like sedimentary rocks, aerogels, and fracture systems exhibit fractal characteristics (Coleman and Vassilicos 2008). Fractals are repeating binary geometrical patterns (either stochastically or exactly) over a range of spatial scales and their properties scale in a power law manner. The popularity of fractals is attributed to Mandelbrot (1982). This influential work created a new geometrical paradigm which was centred on the scaling of heterogeneous systems. *Fractal* came from the Latin word *fractus*, which means fragmented or irregular, so it is not amazing that porous materials have played a significant role in the development and application of fractal models (Perfect *et al.,* 2009). Consequently, porous sediments can be characterized with the use of fractal model or fractal curve that represents the relationship between the radius of pores and number of pores (Li, 2004b). This characterization is important so that the relationship between production rate and time can be established during spontaneous imbibition (Li and Zhao, 2012). On a log-log plot, the fractal curve is a straight line and the slope of the straight line is termed the fractal dimension of the porous sediments. The heterogeneity of the porous medium is represented by the magnitude of fractal dimension. The greater the heterogeneity of the porous material, the greater the fractal dimension (Li, 2004b). The application of fractal modeling of porous media has received enormous attention in the field of reservoir engineering. These applications are namely: the development of capillary pressure models, relative permeability models and models for the prediction of oil production rate (Li and Zhao, 2012). Katz and Thompson (1985) used scanning electron microscopy (SEM) and optical data to show that the pore spaces of many types of sandstone are fractal. They reported a technique to predict rock porosity using the fractal statistics. In 1986, they showed that the percolation concepts could be used to define a characteristic length for the permeability in random porous sediment. There was a quantitative agreement between theory and experiment when the model was applied to sandstone and carbonate rocks with no modifiable parameters. Angulo and Gonzalez (1992) evaluated fractal dimension by utilizing capillary pressure data from mercury intrusion tests. It was targeted that a plot of the volume of mercury intruded into the rock versus capillary pressure would be a power-law function. They proposed a relationship between the scaling exponent and the pore bulk fractal dimensions. Li (2010) used mercury intrusion data to calculate the fractal dimension of the rock from the Geysers geothermal field and showed that the pore systems are fractals with a range of scale of about five orders of magnitude. Li, (2011) again, proposed an interrelationship between resistivity index, capillary pressure, and relative permeability using fractal geometry. Their study showed the fractal features of the distribution of pore size in different porous sediments. Li and Zhao (2012) developed a Fractal Prediction Model by using fractal dimension as one of the parameters to predict production rate by spontaneous imbibition. The model predicts a power law relationship between the rate of spontaneous imbibition and time. Fractal dimension was estimated from the model by using the test data of spontaneous imbibition in porous sediments. They tested the validity of the model by utilizing the test data of recovery in both gas saturated rock and oil saturated rock. From the results, the fractal model matched the experimental data satisfactorily. The data of fractal dimension deduced from the fractal model match were about the same values of fractal dimension obtained using mercury-intrusion capillary pressure technique in a porous media. Gao *et al.*, (2014) developed a fractal model to explain the pore structure of reservoirs with similar lithology based on the existing dimensionless capillary pressure function $J(S_W)$ and the theory of fractal geometry. They determined that when the parameters of the microstructures (that is, the ratio of the maximum to the minimum pore diameter, fractal dimension for tortuosity and fractal dimension for pore area) are equal or close respectively for porous core samples (with differing wetting angles, permeabilities, interfacial tensions and porosities), the $J(S_W)$ function values vs. saturation coincide with one another for wetting phase. They found that a similar lithology has the same values of the maximum pore diameter/ the minimum pore diameter ($\lambda_{max}/\lambda_{min}$), fractal dimension for tortuosity of capillaries (D_τ) and fractal dimension for pore area (D_f). They tested the validity of the model with the experimental data of Brown, (1951) and that of Slider, (1976). They concluded that the predictions from the fractal expression of the capillary pressure function $J(S_W)$ strongly agree with the experimental data. The model had a better fit for the experimental data from Slider than the data from Brown. The above studies demonstrate the capability of the subject of fractal geometry and how they can be applied in the industry. Many of the above stated work are centred on vertical spontaneous imbibition. Based on this, we derived an analytical expression for horizontal spontaneous imbibition to calculate the accumulated imbibition weight of liquid imbibed into a porous medium in gas/water/rock systems by taking into account the fractal characters of the pores.

Mathematical Formulation of Fractal Model for Spontaneous Imbibition

In this study, the following assumptions are made: spontaneous imbibition occurred horizontally, porous media are composed of a bundle of tortuous capillaries/channels with varying cross-sectional area, and there is statistical self-similarity in the range of the minimum to the maximum pore diameter in porous sediment. The cumulative size distribution of pores in a porous medium follows a fractal scaling law (Mandelbrot, 1982; Jiang *et al*., 2013):

$$N(\geq \lambda)= \left(\frac{\lambda_{max}}{\lambda}\right)^{D_f} \quad 2.1$$

where N is the total number of pores whose sizes equal to and greater than λ, D_f is the fractal dimension for pores, D is pore diameter and L is the length scale. The number of pores in a fractal porous rock, whose sizes range from λ to $\lambda + d\lambda$ is expressed as (Cai *et al*., 2010a):

$$-dN = D_f\ \lambda_{max}^{D_f}\ \lambda^{-(D_f+1)}\ d\lambda \qquad 2.2$$

and the pores probability density function is expressed as:

$$f(\lambda) = D_f\ \lambda_{min}^{D_f}\ \lambda^{-(D_f+1)} \qquad 2.3$$

where λ is the pore diameter and $\lambda_{min} \leq \lambda \leq \lambda_{max}$, λ_{min} and λ_{max} are the minimum and maximum pore diameter, the fractal dimension D_f ranges from $0 < D_f < 2$ in two-dimensional (2D) plane and $0 < D_f < 3$ in three dimensional (3D) space. Jiang *et al*., (2013) stated that when water flow through the pores of porous medium, the capillaries become tortuous and these tortuous capillaries are expressed by fractal equation as:

$$L_a(\lambda) = L_o^{D_\tau} \lambda^{(1-D_\tau)} \qquad 2.4$$

where D_τ is the fractal dimension for tortuous capillaries and it is in the range of 1 □ D_τ □ 2 in two-dimensional (2D) and $0 < D_\tau < 3$ in three dimensional (3D) planes respectively, representing the degree of convolutedness of capillary track for the flow of fluid through a porous sediment. $L_a(\lambda)$ is the tortuous(actual) length along the flow path (that is, the actual distance imbibed by the fluid), L_o is the representative(straight) length of tortuous flow path. $D_\tau = 1$ for a straight capillary pathway and for a highly tortuous capillary, a higher value of D_τ is obtained. For a straight capillary $L_a(\lambda) = L_o$. Cai and Sun (2013) defined tortuousity (τ) of the tortuous capillary as:

$$(\tau) = \frac{L_a(\lambda)}{L_o} = \left(\frac{L_o}{\lambda}\right)^{D_\tau - 1} \qquad 2.5a$$

and $D_\tau = 1 + \frac{In[(\tau_a\{D_f + D_\tau - 1\})/D_f]}{In(L_o/\lambda_{min})}$ 2.5b

τ_a represents the average tortuosity of the tortuous capillaries. The total pore cross-sectional area in porous media can be computed by the equation (Cai and Sun, 2013; Cai et al., 2012):

$$A_p = \frac{\pi\lambda_{max}^2}{4}\frac{D_f}{2-D_f}(1-\emptyset) \qquad 2.6$$

The whole cross-sectional area of the fractal unit is then calculated by:

$$A_u = \frac{\pi\lambda_{max}^2}{4}\frac{D_f}{2-D_f}\frac{1-\emptyset}{\emptyset} = \frac{A_p}{\emptyset} \qquad 2.7$$

where $\emptyset$ is the porosity. The cross-sectional areas of tortuous capillaries in a natural porous media vary. Therefore, in a cross-sectional area A_f, the cumulative number N_f of pores is given by:

$$N_f(\geq \lambda) = \frac{A_f}{A_u} N = \frac{A_f}{A_u}\left(\frac{\lambda_{max}}{\lambda}\right)^{D_f} \qquad 2.8$$

Substituting eq. (2.7) into eq. (2.8) yields:

$$N_f(\geq \lambda) = \frac{4A_f}{\pi\lambda_{max}^{2-D_f}}\frac{2-D_f}{D_f}\frac{\emptyset}{1-\emptyset}\lambda^{-D_f} \qquad 2.9$$

To obtain the number of pore sizes lying between λ and $d\lambda$ in the area A_f, we differentiate eq. (2.9):

$$-dN_f = \frac{4A_f}{\pi\lambda_{max}^{2-D_f}}\frac{(2-D_f)\emptyset}{1-\emptyset}\lambda^{-(D_f+1)}d\lambda \qquad 2.10$$

The negative sign in eq. (2.10) indicates that as the pore size increases the number of pores decreases. For horizontal spontaneous imbibition, the imbibition flow rate is given as:

$$q(\lambda) = \frac{\pi}{128}\frac{\lambda^4}{\mu L_a}\left(\frac{4\sigma cos\theta}{\lambda} + g\rho L_z\right) \qquad 2.11$$

If the spontaneous imbibition occurs in a piston-like fashion for homogeneous porous medium, the total water flow rate imbibed into the sediment is calculated by the integration of all the flow rates:

$$Q = -\int_{\lambda_{min}}^{\lambda_{max}} q(\lambda)\, dN_f = \frac{\sigma cos\theta}{8\mu L_a((\lambda)}\frac{2-D_f}{3-D_f}\frac{A_f\emptyset\lambda_{max}}{1-\emptyset} + \frac{g\rho L_o}{64\mu L_a(\lambda)}\frac{2-D_f}{4-D_f}\frac{A_f\emptyset\lambda_{max}^2}{1-\emptyset} \qquad 2.12$$

The average imbibition velocity is thus obtained by the division of eq. (2.12) by the area of the pore:

$$\square_a = \frac{\sigma cos\theta}{8\mu L_a((\lambda)} \frac{2-D_f}{3-D_f} \frac{\lambda_{max}}{1-\emptyset} + \frac{g\rho L_o}{64\mu L_a(\lambda)} \frac{2-D_f}{4-D_f} \frac{\lambda_{max}^2}{1-\emptyset}$$
2.13

The increase in weight due to the action of imbibition is thus, given as:

$$M = \rho A_f \emptyset L_o \quad 2.14$$

If we differentiate eq. (2.14) with respect to time t, we have:

$$\frac{dM}{dt} = \rho A_f \emptyset v_o \quad 2.15$$

where $v_o = \square_a / \tau_a$. Substituting eq. (2.13) into eq. (2.15) results in:

$$\frac{dM}{dt} = \left(\rho A_f \emptyset\right)^2 \left[\frac{\sigma cos\theta}{8\mu \tau_a^2} \frac{2-D_f}{3-D_f} \frac{\lambda_{max}}{1-\emptyset} + \frac{g\rho L_o}{64\mu \tau_a^2} \frac{2-D_f}{4-D_f} \frac{\lambda_{max}^2}{1-\emptyset}\right] \frac{1}{M} \quad 2.16$$

Solving eq. (2.16) and rearranging gives:

$$M = \left(\rho A_f \emptyset\right) \left[\frac{\sigma cos\theta}{8\mu \tau_a^2} \frac{2-D_f}{3-D_f} \frac{\rho A_f \emptyset \lambda_{max}}{1-\emptyset} + \frac{A_f g \rho^2 L_o}{64\mu \tau_a^2} \frac{2-D_f}{4-D_f} \frac{\emptyset \lambda_{max}^2}{1-\emptyset}\right]^{1/2} t^{1/2} \quad 2.17$$

The proposed analytical model (Eq. 2.17) for spontaneous imbibition of wetting fluid into a porous rock indicates that the accumulated weight of liquid imbibed into porous medium is proportional to $t^{1/2}$ in the whole imbibition process. Obviously, from eq. (2.17) the factors that govern the spontaneous imbibition comprises: L_o (the straight length of tortuous flow path), D_f, λ_{max} ,$\emptyset$, τ_a and A_f (the microstructural parameters of porous sediment), μ, ρ, σ (properties of the fluid) and θ (the fluid-solid interaction). However, existing imbibition models by conventional approaches such as thee Handy model, Li and Horne model cannot disclose the microstructural parameters of porous sediments. Thus eq. (2.17) provides a better understanding of spontaneous imbibition mechanisms. Two constants exist in eq. (2.17):

$$a = \frac{\sigma cos\theta}{8\mu \tau_a^2} \frac{2-D_f}{3-D_f} \frac{(\rho A_f \emptyset)^2 \lambda_{max}}{1-\emptyset} \text{ and } b = \frac{A_f^2 g \rho^3 L_o}{64\mu \tau_a^2} \frac{2-D_f}{4-D_f} \frac{\emptyset^2 \lambda_{max}^2}{1-\emptyset} \quad 2.18$$

Therefore, $\frac{dM}{dt} = \frac{a}{M} + b$ 2.19

To determine the equilibrium imbibition weight, M_e, we set $\frac{dM}{dt} = 0$:

$$M_e = -\frac{a}{b} = -\frac{8\sigma cos\theta}{\lambda_{max} L_o} \frac{4-D_f}{3-D_f g\rho} \quad 2.20$$

But the minimum capillary pressure, $P_{c\,min} = \frac{4\sigma cos\theta}{\lambda_{max}}$ (Cai *et al*., 2014), implies:

$$M_e = -P_{c\,min} \frac{2}{L_o g\rho} \frac{4-D_f}{3-D_f} \quad 2.21$$

Eq. (2.20) indicates that the equilibrium weight is dependent on D_f, λ_{max} (the structural parameters), ρ, σ (properties of the fluid), θ (the fluid-solid interaction), g (the gravitational acceleration) and L_o (the straight length of tortuous flow path).

Results and Discussions

It can be seen from the analytical model, (eq. 2.17) that the accumulated weight of liquid that imbibed into a porous material is a function of the straight length of tortuous flow path, the structural parameters of porous material, properties of the fluid and the fluid-solid interaction. To compute spontaneous imbibition weight using eq. (2.17), the parameters, D_f, λ_{max} , τ_a are derived using theoretical models while L_o

,$\emptyset$, A_f, μ, ρ, σ and θ are obtained from experimental measurements. The pore fractal dimension (D_f) can be computed by the expression (Yu and Li, 2001; Cai *et al*., 2010b; Zheng *et al*., 2013):

$$D_f = \text{d} - \frac{In\emptyset}{In\left[\frac{\lambda_{min}}{\lambda_{max}}\right]} \qquad 2.22$$

where $\emptyset$ is porosity, d is the Euclidean dimension (d = 2 is used in this study) . To determine the maximum pore diameter, the model of square arrangement of circular particles (Wu and Yu, 2007) is used:

$$\lambda_{max\,1} = \frac{D_s}{2}\left[\sqrt{\frac{\emptyset}{1-\emptyset}} + \sqrt{\frac{\pi}{4(1-\emptyset)}} - 1\right] \qquad 2.23a$$

The maximum pore diameter can as well be determined based on the model of an equilateral triangle arrangement of circular particles (Yu and Cheng, 2002):

$$\lambda_{max\,2} = \frac{D_s}{2}\left[\sqrt{\frac{2\emptyset}{1-\emptyset}}\right] \qquad 2.23b$$

The average maximum pore diameter λ_{max} is thus obtained by the expression:

$$\lambda_{max} = \frac{\lambda_{max\,1}+\lambda_{max\,2}}{2} \qquad 2.23c$$

The characteristic diameter of particles (D_s) can be calculated based on the rearranged Kozeny-Carman equation (Carman, 1937):

$$D_s = \frac{4(1-\emptyset)}{\emptyset}\sqrt{\frac{Kk}{\emptyset}} \qquad 2.24$$

where K is permeability and k is Kozeny constant which considers tortuosity of capillaries and pore non-uniformity and is equal to 4.8 ± 0.3. The average tortuousity of flow path can be determined by (Comiti and Renaud, 1989):

$$\tau_a = 1 + 0.41\text{n}\frac{1}{\emptyset} \qquad 2.25$$

where 0.41 is a factor obtained when experimental data were fitted for spherical particles. Once $\emptyset$ and D_s are known, the parameters D_f, λ_{max} and τ_a are computed using equations (2.22), (2.23c) and (2.25) respectively. The accumulated weight of liquid imbibed into the porous material is then obtained from proposed analytical model, eq. (2.17). In this work water was considered as the imbibing liquid displacing air in order to numerically analyze the effect of the structure of the porous medium. The relevant parameters used are σ = 72mN/m, μ = 1.0 mpas, A_f = 3cm^2, ρ = 1.0g/cm^2, θ = 0^o $\lambda_{min}/\lambda_{max}$ = 0.01, D_s = 0.02cm.

Figure 1shows the accumulated weight of water imbibed into the porous medium verses square root of time for different values of porosity using the above stated parameters of the model (eq. 2.17). It can be seen that lower porosity requires longer time to reach equilibrium weight. It can be deduced from the figure that as the imbibition weight increases, the rate of imbibition also decreases gradually. This phenomenon occurs because the driving force was gradually consumed as the water is penetrating forward. Again from the figure, there is an increase in imbibition weight with porosity at fixed time. This is because the pore area of the medium exposed to water increases with increase in porosity, thus allowing more water to be imbibed into the porous material. When there is a decrease in porosity the spontaneous imbibition rate decreases. When the porosity is low, definitely the permeability will be lower, meaning smaller pore throats. Figure 2 shows the effect of pore fractal dimension on Horizontal spontaneous imbibition. The imbibition weight increases with increase in pore fractal dimension. This is because as the porosity increases, the pore fractal dimension also increases and the pore area of the porous medium exposed to water increases. The validity of the proposed model (eq. 2.17), is tested using the experimental data of Huber *et al*., 2007. The model predictions for imbibition weight were compared with the experimental results and they are shown in Figure 3. The experiment was carried out at 23^{o}C with water as the imbibing fluid into meso-porous silica. The following measurements: (= 72.25mN/m, μ = 0.95 mpa s, ρ = 1.0g/cm^2, θ = 0^o, D_s = 25.4nm, A_f = 0.4cm x 0.43cm and average pore diameter of 5nm) were used to fit the experimental data which gave a perfect match between the observed and the computed imbibition water weight versus time.

Conclusion

In this study, an analytical model for horizontal spontaneous imbibition of wetting fluid into gas-saturated porous medium over the entire imbibition process has been proposed based on fractal geometry by considering the effect of capillary pressure and gravity force. The accumulated imbibition weight computed by the proposed fractal model (eq. 2.17) depends on pore fractal dimension D_f, Area A_f, porosity $\emptyset$, liquid-solid interaction θ, maximum pore diameter λ_{max}, and the liquid properties (μ, ρ, σ). The results of the model show that the accumulated weight follows $t^{1/2}$ scaling law in the entire imbibitions process. This phenomenon is only noticed in the initial stage of vertical spontaneous imbibition (Cai and Sun, 2013). The derived analytical expression when compared with published experimental data is in agreement with them. The model is also consistent with previously published work.

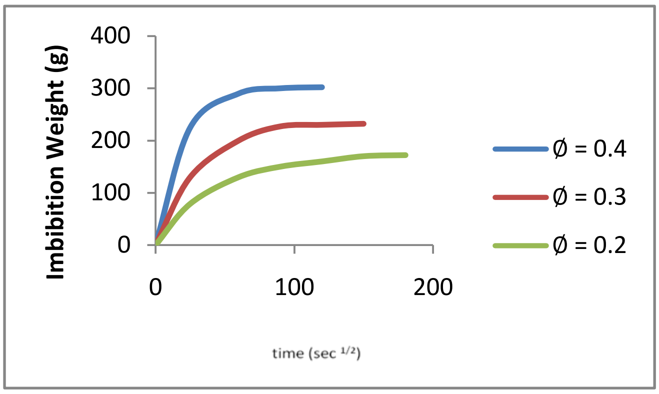

Fig. 1: Accumulated weight of water imbibed into the porous medium verses square root of time.
The above stated parameters are used in eq. (32)

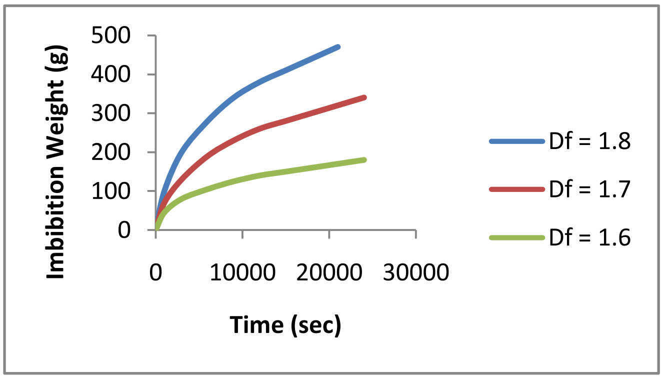

Fig. 2: Accumulated weight of water imbibed into the porous medium verses time at different fractal dimension.

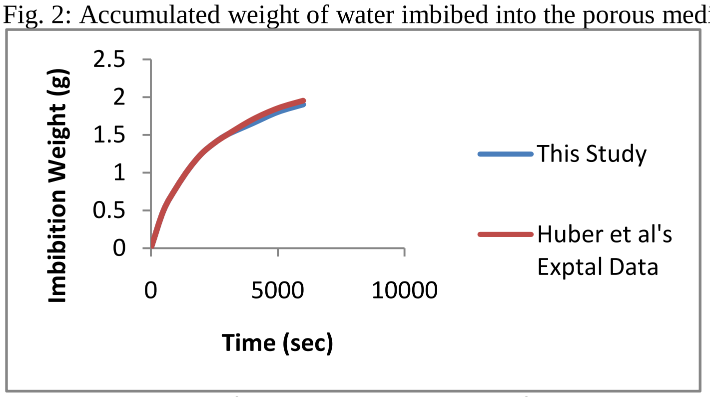

Fig. 3: Comparison of the accumulated weight of water imbibed into the porous medium verses time determined by the proposed model (eq. 32) with the experimental data in Meso-porous silica (Huber *et al.,* 2007).

References

[1]. Al-Lawati, Shabir and Saleh, Saad (1996): Oil Recovery in Fractured Oil Reservoirs by Low IFT Imbibition Process. Paper SPE 36688, paper presented at the 1996 SPE Annual Techmcal Conference and Exhibition held in Denver, Colorado, U S.A. 6– 9 October: 107-118.

[2]. Angulo, R.F., Gonzalez, H. (1992): Fractal dimensions from mercury intrusion capillary tests. SPE 23695 presented at the 2nd Latin American petroleum engineering conference of SPE held in Caracas, Venezuela, 8–11 March.

[3]. Babadagli, T. (2001): Scaling of Cocurrent and Countercurrent Capillary Imbibition for Surfactant and Polymer Injection in Naturally Fractured Reservoirs. SPE Journal, December: 465-478.

[4]. Benavente, D., Lock, P., Angeles García Del Cura, M. and Ordonez, S. (2002): Predicting the capillary imbibition of porous rocks from microstructure. Transp. Porous Media (49): 59−76.

[5]. Brown, H.W. (1951): Capillary pressure investigations, Journal of Petroleum Technology 3 (3): 67–74.

[6]. Cai, Jianchao, Yu, Boming Zou, Mingqing and Luo, Liang (2010): Fractal Characterization of Spontaneous Co-current Imbibition in Porous Media. Energy Fuels (24): 1860–1867.

[7]. Cai, Jianchao, Yu, Boming Zou, Mingqing and Mei Maofei (2010): Fractal analysis of invasion depth of extraneous fluids in porous media. Chemical Engineering Science (65): 5178–5186.

[8]. Cai, J., Hu, X., Standnes, D. C. and You, L. (2012): An analytical Model for Spontaneous Imbibition in Fractal Porous Media including Gravity. Colloids and Surfaces A: Physicochem. Eng. Aspects (414):228-233.

[9]. Cai Jianchao, and Sun Shuyu (2013): Fractal Analysis of Fracture increasing Spontaneous Imbibition in Porous Media with Gas Saturated. Internal Journal of Modern Physics 24(8): 1-13.

[10]. Carman, P. C. (1937): Fluid flow through granular beds. Trans. Inst. Chem. Eng. (15): 150–167.

[11]. Coleman, S.W. and Vassilicos, J. C. (2008): Transport Properties of Saturated and Unsaturated Porous Fractal Materials. Physical Review Letters (100): 1-4.

[12]. Comiti, J. and Renaud, M. (1989): A new model for determining mean structure parameters of fixed beds from pressure drops measurements: Application to beds packed with parallel epipedal particles. Chem. Eng. Sci. 44 (9):1539–1545.

[13]. Fernø, M. A. (2012): "Introduction to Enhanced Oil Recovery (EOR) Processes and Bioremediation of Oil-Contaminated Sites", 1st Ed. Intech Publishing, p 89-119.

[14]. Gao Hongjing, Yu Boming, Duan Yonggang, Fang Quantang (2014): Fractal analysis of dimensionless capillary pressure function. International Journal of Heat and Mass Transfer (69): 26–33.

[15]. Handy, L.L. (1960): Determination of Effective Capillary Pressures for Porous Media from Imbibition Data, Petroleum Transactions AIME, Vol. 219: 75-80.

[16]. Høgnesen, E. J., Standnes, D. C. and Austad, T. (2004): Scaling spontaneous imbibition of aqueous surfactant solution into preferential oil-wet carbonates. Energy Fuels 18(6): 1665–1675.

[17]. Huber, P., Gruner, S., Schafer, C., Knorr, K. and Kityk, A. V. (2007): Rheology of liquids in nanopores: A study on the capillary rise of water, n-Hexadecane and n-Tetracosane in mesoporous silica. Eur. Phys. J. Spec. Top. (141): 101−105.

[18]. Jiang, Guoping, Shi Wei, and Huang Lili (2013): Fractal Analysis of Permeability of Unsaturated Fractured Rocks. Scientific World Journal, Vol. 2013: 1-5.

[19]. Katz, A.J. and Thompson, A. H. (1985): Fractal Sandstone Pores: Implications for Conductivity and Pore Formation. Phys. Rev. Lett. (54): 1325-1328.

[20]. Katz, A.J. and Thompson, A. H. (1986): Quantitative prediction of permeability in porous rock. Phy. Rev. B 34(11): 8179–8181.

[21]. Krohn, C. E. (1988): Fractal Measurements of Sandstones, Shales, and Carbonates. Journal of Geophysical Research 93(B4): 3297-3305.

[22]. Lefebvre Du Prey, E. (1978): Gravity and Capillarity Effects on Imbibition in Porous Media. Paper SPE 6192, published in SPE Journal (18): 195-206.

[23]. Laughlin, R. D., Davies, J. E. (1961): Some aspects of capillary absorption in fibrous textile wicking. Textile Res. U. (31): 39.

[24]. Li, Kewen and Horne, Roland N. (2001): Characterization of Spontaneous Water Imbibition into Gas-Saturated Rocks. SPE Journal, 6(4): 375-384.

[25]. Li, K. (2004a): Characterization of Rock Heterogeneity Using Fractal Geometry. Paper SPE 86975, presented at the Proceedings of the 2004 SPE Western Region Meeting, Bakersfield, CA, USA, March 16-18.

[26]. Li, Kewen (2004b): Theoretical Development of the Brooks-Corey Capillary Pressure Model from Fractal Modeling of Porous Media. Paper SPE 89429 presented at the 2004 SPE/DOE Fourteenth Symposium on Improved Oil Recovery held in Tulsa, Oklahoma, U.S.A., 17–21 April: 1-6.

[27]. Li, K. (2010): Analytical derivation of Brooks-Corey capillary pressure model using fractal geometry and evaluation of rock heterogeneity. Journal Petroleum Science and Engineering 73(1–2): 20–26.

[28]. Li, K. (2011): Interrelationship between resistivity index, capillary pressure and relative permeability. Transport in Porous Media**.** 88(3): 385-398.

[29]. Li, Kewen and Zhao, Haiyang (2012): Fractal Prediction Model of Spontaneous Imbibition Rate. Transp Porous Med (91): 363–376.

[30]. Lucas, R. (1918): Rate of capillary ascension of liquids. Colloid Polymer Science 23(1): 15−22.

[31]. Mandelbrot, B. B. (1982): The fractal geometry of nature. W. H. Freeman, New York.

[32]. Perfect, E. Pachepsky, Y. and Martin, M. A. (2009): Fractal and Multifractal Models Applied to Porous Media. vadose zone journal 8 (1): 174-176.

[33]. Schembre, J. M., Akin, S., Castanier, L. M. and Kovscek, A. R. (1998): Spontaneous Water Imbibition into Diatomite. Paper SPE 46211, presented at the 1998 Western Regional Meeting held in Bakersfield, California, 10-13 May: 1-11.

[34]. Slider, H.C. (1976): Practical Petroleum Reservoir Engineering Methods, Petroleum Publishing Company, University of Michigan.

[35]. Standnes, Dag Chun (2009): Calculation of Viscosity Scaling Groups for Spontaneous Imbibition of Water Using Average Diffusivity Coefficients. Energy and Fuels (23): 2149-2156.

[36]. Tavassoli, Z., Zimmerman, R.W. and Blunt, M. J. (2005): Analytic analysis for oil recovery during counter-current imbibition in strongly water-wet systems. Transp. Porous Med. (58): 173–189.

[37]. Toledo, P. G., Novy, R. A., Davis, H. T., and Scriven, L. E. (1993): Capillary Pressure, Water Relative Permeability, Electrical Conductivity and Capillary Dispersion Coefficient of Fractal Porous Media at Low Wetting Phase Saturation. SPE Advanced Technology Series 2(1): 136-141.

[38]. Washburn, E.W. (1921): The dynamics of capillary flow. Phys. Rev. (17): 273–283.

[39]. Wu, J. S. and Yu, B. M. (2007): A fractal resistance model for flow through porous media. International Journal of Heat Mass Transfer 50 (19-20): 3925–3932.

[40]. Yu, B. M. and Cheng, P. (2002): A fractal permeability model for bi-dispersed porous media. International Journal of Heat Mass Transfer 45 (14): 2983–2993.

[41]. Yu, B. M. and Li, J. H. (2001): Some fractal characters of porous media. Fractals 9 (3): 365–372.

[42]. Zhang, X., Morrow, N. R. and Ma, S. (1996): Experimental Verification of a Modified Scaling Group for Spontaneous Imbibition. SPE Reservoir Engineering (10): 280-285.

[43]. Zheng Qian, Yu Boming, Duan Yonggang and Fang Quantang (2013): A fractal model for gas slippage factor in porous media in the slip flow regime. Chemical Engineering Science (87): 209–215.

[44]. Zimmerman, R. W. and Bodvarsson, G. S. (1991): A simple approximate solution for horizontal infiltration in a Brooks-Corey medium. Trans. Porous Med. (6): 195–205.

