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# Analysis of Successive Occurrence of Digit 4 in Prime Numbers till 1 Trillion

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**Abstract:** Taking high limit of 1 trillion, all prime numbers till that are analyzed for successive occurrence of 4 in their digits. All possible multiple successive occurrences of 4's are searched. The number of primes in different ranges of increasing powers of 10 with multiple successive 4's in their digits are compared exhaustively with those containing equal number of successive 1, 2 and 3 are compared. Within all first 12 ranges of $1 - 10^n$, the smallest and the largest primes with multiple successive digit 4's in them are also determined.
**Keywords:** Prime numbers, digit 4, successive occurrences
**Mathematics Subject Classification 2010:** 11Y35, 11Y60, 11Y99.

## I. INTRODUCTION

Numbers have been one of the very first inventions of human kind, although independently in many continents across the globe and with different perspectives [2]. Special of them like prime number still maintain their charm.

Even today primes await their fitting into a simplistic formula owing to which the mathematician are left with two approaches to explore them: one covering almost all primes but allowing approximations [1] and the other precise deterministic approach coming at the cost of limitedness in addressability to though huge but limited number of them [4].

The all, general, and non-successive occurrences of 0 and those of non-zero digits like 4 in all natural numbers are can be inferred from analysis in [5], [6], [7] and [11], [12], [13], respectively. Its occurrence in prime numbers is also analyzed [23]. Similar investigation of general, successive and non-successive occurrences of digit 0, 1, 2 and 3 in primes can be found in [8]-[10], [14]-[22].

This work is about successive 4's in digits of primes.

## II. OCCURRENCE OF SINGLE SUCCESSIVE DIGIT 4 IN PRIME NUMBERS

As single occurrence of any digit is trivially successive, values computed in [23] for occurrences of single 4 in digits of primes are equally occurrences of single successive 4 in them!

**Table 1**[23] : Number of Prime Numbers in Various Ranges with Single (Successive!) 4 in Their Digits

| Sr. No. | Range | Number of Primes with Single Successive(!) 4 |
|---|---|---|
| 1. | $1 - 10^1$ | 0 |
| 2. | $1 - 10^2$ | 3 |
| 3. | $1 - 10^3$ | 30 |
| 4. | $1 - 10^4$ | 294 |
| 5. | $1 - 10^5$ | 2,725 |
| 6. | $1 - 10^6$ | 25,602 |
| 7. | $1 - 10^7$ | 234,745 |
| 8. | $1 - 10^8$ | 2,142,049 |
| 9. | $1 - 10^9$ | 19,446,059 |
| 10. | $1 - 10^{10}$ | 176,268,251 |
| 11. | $1 - 10^{11}$ | 1,595,405,886 |
| 12. | $1 - 10^{12}$ | 14,425,647,017 |

All results here are outcomes of multiple computers crunching numbers parallely and continuously using excellent algorithms [3].

## III. OCCURRENCE OF MULTIPLE SUCCESSIVE 4'S IN PRIME NUMBERS

Now let's inspect 2 and more successive occurrences of 4's, as successivity will have distinctive meaning in these cases. Using formula in [12], the number of all natural numbers containing multiple successive 4's in their digits within the ranges of $1 - 10^n$, $1 \le n \le 12$ are known. This we do for primes here; although by direct computation rather than a formula as it is just not available!

**Table 2**: Number of Primes in Various Ranges with Multiple Successive 4's in Their Digits

| Sr. No. | Number Range < | Number of Primes with 2 Successive 4's | Number of Primes with 3 Successive 4's | Number of Primes with 4 Successive 4's |
|---|---|---|---|---|
| 1. | $10^3$ | 2 | 0 | 0 |
| 2. | $10^4$ | 21 | 2 | 0 |
| 3. | $10^5$ | 241 | 18 | 1 |
| 4. | $10^6$ | 2,269 | 201 | 12 |
| 5. | $10^7$ | 21,795 | 1,972 | 152 |
| 6. | $10^8$ | 203,489 | 18,955 | 1,672 |
| 7. | $10^9$ | 1,889,934 | 179,150 | 16,691 |
| 8. | $10^{10}$ | 17,404,886 | 1,691,189 | 160,892 |
| 9. | $10^{11}$ | 159,535,676 | 15,756,962 | 1,530,845 |
| 10. | $10^{12}$ | 1,457,037,501 | 145,695,925 | 14,383,476 |

**Table 3**: Continued …

| Sr. No. | Number Range < | Number of Primes with 5 Successive 4's | Number of Primes with 6 Successive 4's | Number of Primes with 7 Successive 4's |
|---|---|---|---|---|
| 1. | $10^6$ | 2 | 0 | 0 |
| 2. | $10^7$ | 16 | 0 | 0 |
| 3. | $10^8$ | 153 | 11 | 0 |
| 4. | $10^9$ | 1,499 | 130 | 7 |
| 5. | $10^{10}$ | 15,017 | 1,324 | 109 |
| 6. | $10^{11}$ | 145,728 | 13,431 | 1,149 |
| 7. | $10^{12}$ | 1,397,959 | 133,159 | 12,283 |

**Table 4**: Continued …

| Sr. No. | Number Range < | Number of Primes with 8 Successive 4's | Number of Primes with 9 Successive 4's | Number of Primes with 10 Successive 4's | Number of Primes with 11 Successive 4's |
|---|---|---|---|---|---|
| 1. | $10^9$ | 1 | 0 | 0 | 0 |
| 2. | $10^{10}$ | 7 | 1 | 0 | 0 |
| 3. | $10^{11}$ | 108 | 8 | 1 | 0 |
| 4. | $10^{12}$ | 1,128 | 97 | 6 | 1 |

Now these values when plotted graphically with logarithmic vertical axis show following patterns.

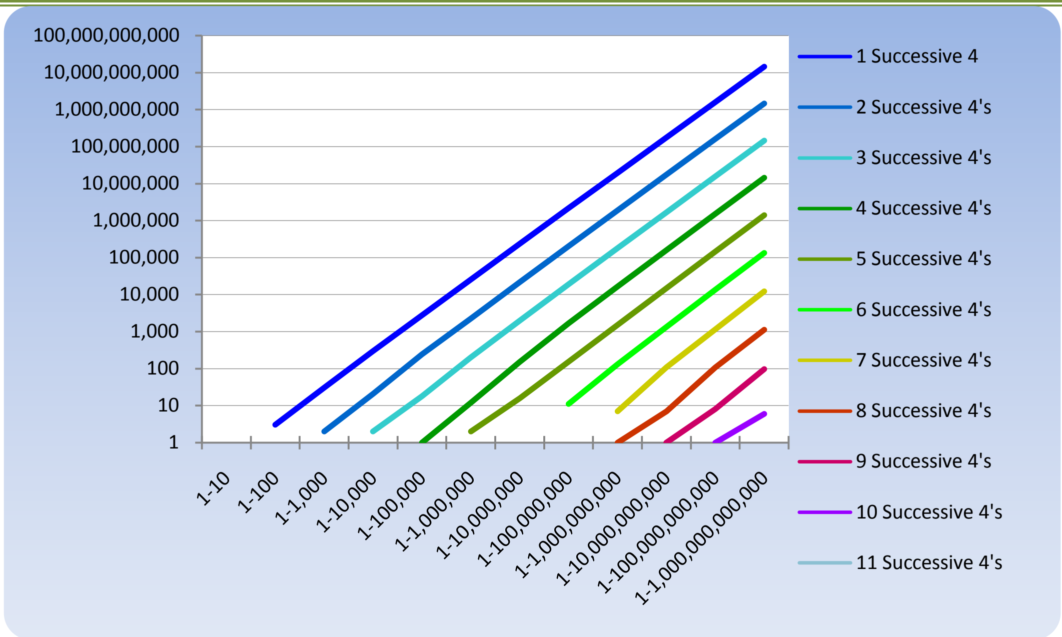



**Figure 1**: Number of Primes in Different Ranges Containing Multiple Successive 4's in Their Digits

The percentage of number of prime numbers having different number of successive 4's in their digits with respect to number of all natural numbers with equal of those in corresponding ranges is quite less.

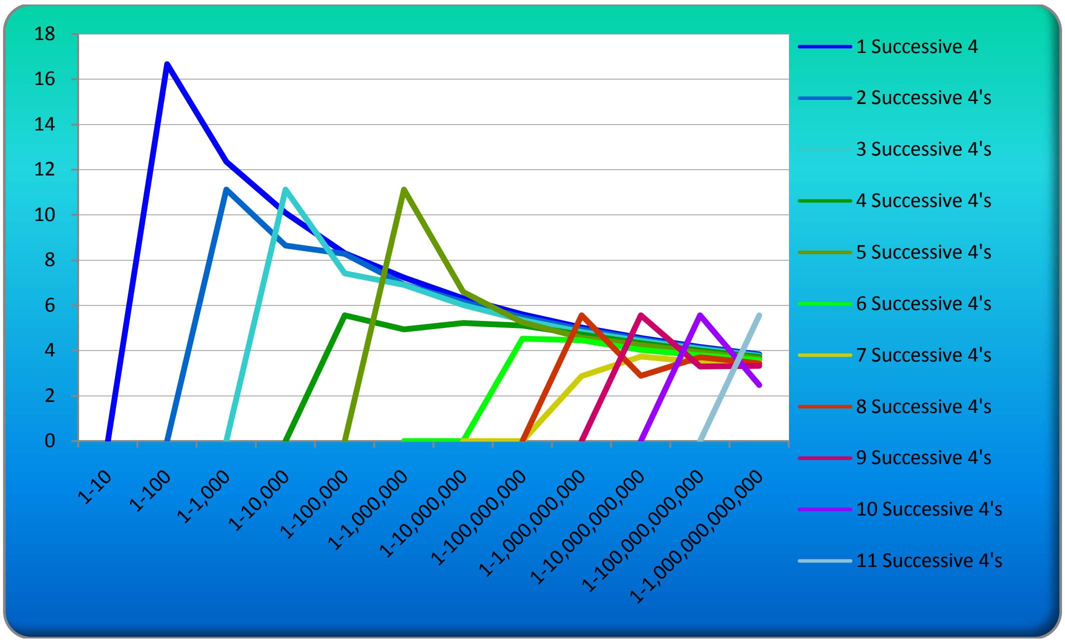



**Figure 2**: Percentage of Primes in Different Ranges with Multiple Successive 4's in Their Digits with Respect to All Such Integers in Respective Ranges

Differences of number of multiple successive occurrences of digits 1, 2 and 3 in primes with those of 4 in them in our ranges are depicted below graphically by dividing them in two blocks – one with 1 and 3 and the other with 4; the former ones can occupy units place and the later one cannot (except in unique case of 2). Digit 0 is not considered because it doesn't occupy units and leading $n^{th}$ places in any $n$ digit prime number.

Of these graph sets, the first given in Figures 3 and 4 for comparison of one digit are essentially same as that in [23] because, as commented earlier, single occurrence is deemed to be successive.

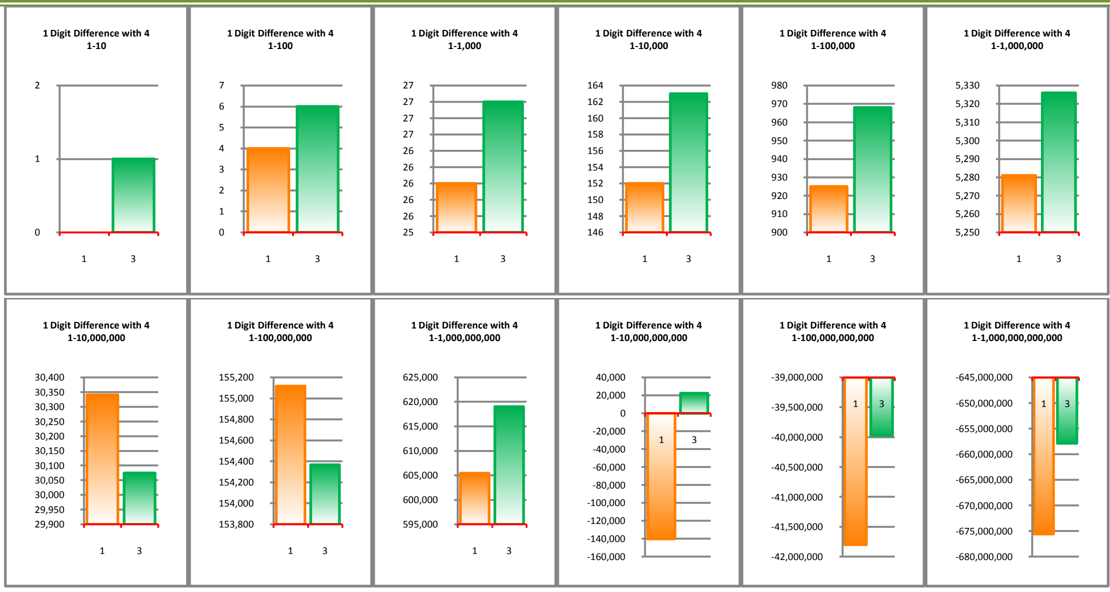



**Figure 3:** Differences of Number of Primes having One(Successive) 1 and One3 in their Digits with those having One(Successive) 4 in them in Ranges of 1 – $10^n$.

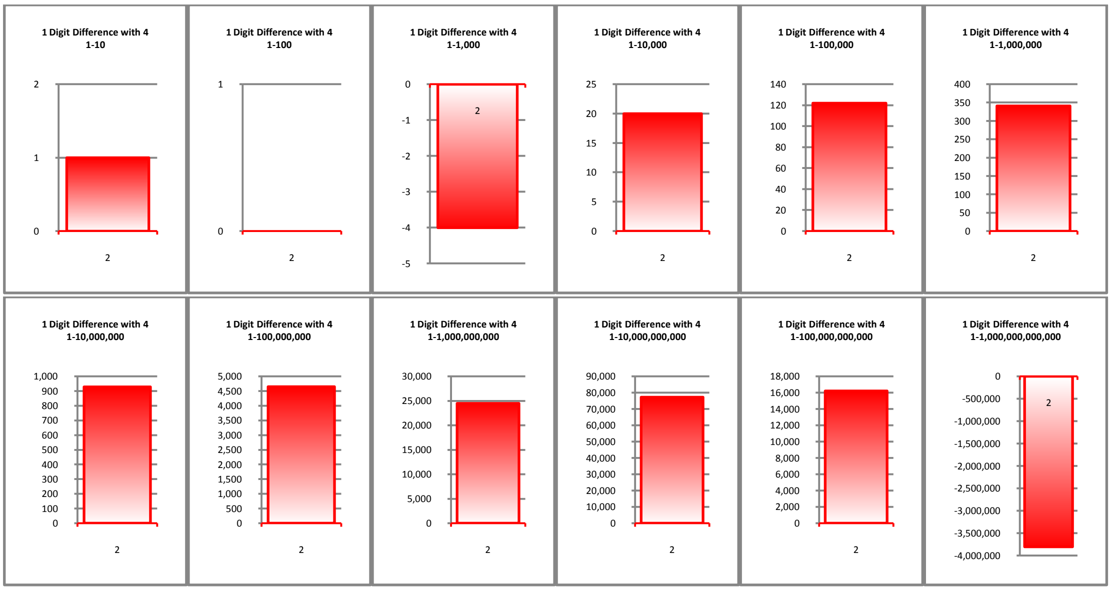
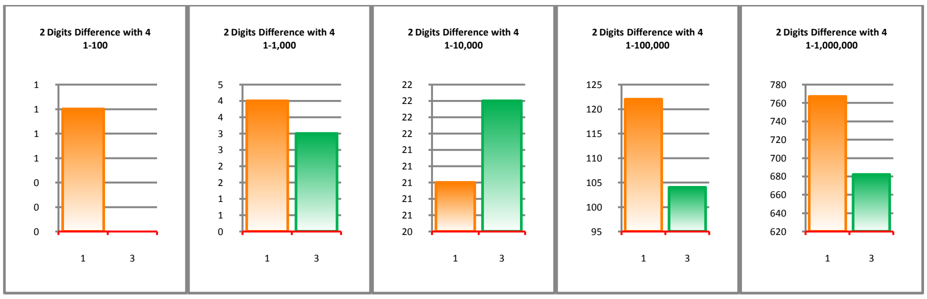



**Figure 4:** Difference of Number of Primes having One(Successive) 2 in their Digits with those having One(Successive) 4 in them in Ranges of 1 – $10^n$.



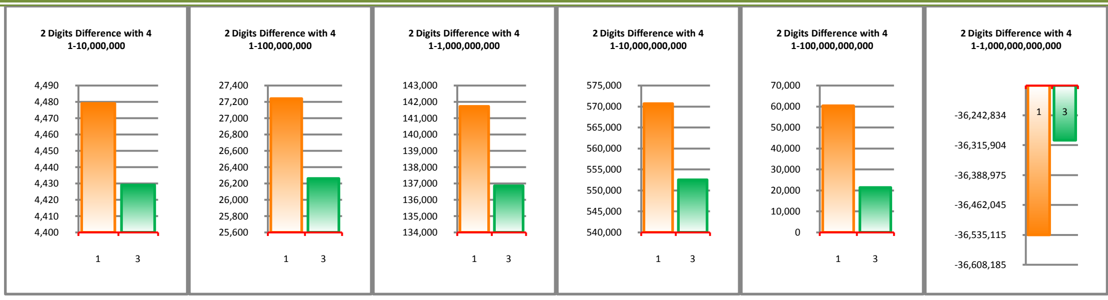



**Figure 5:** Differences of Number of Primes having TwoSuccessive 1's and 3's in their Digits with those having TwoSuccessive 4's in them in Ranges of $1 - 10^n$.

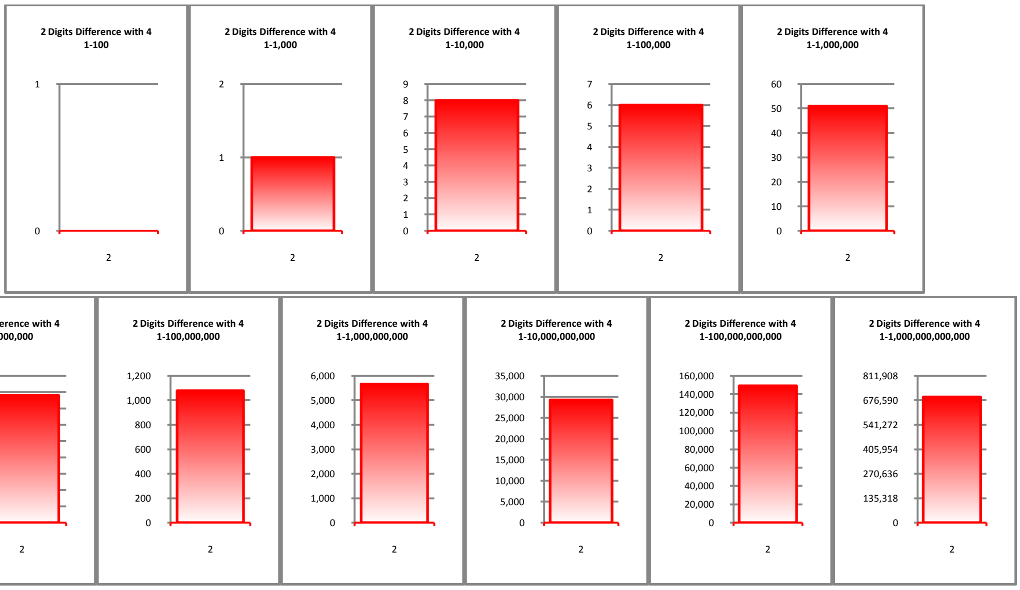



**Figure 6:** Difference of Number of Primes having TwoSuccessive 2's in their Digits with those having TwoSuccessive 4's in them in Ranges of $1 - 10^n$.

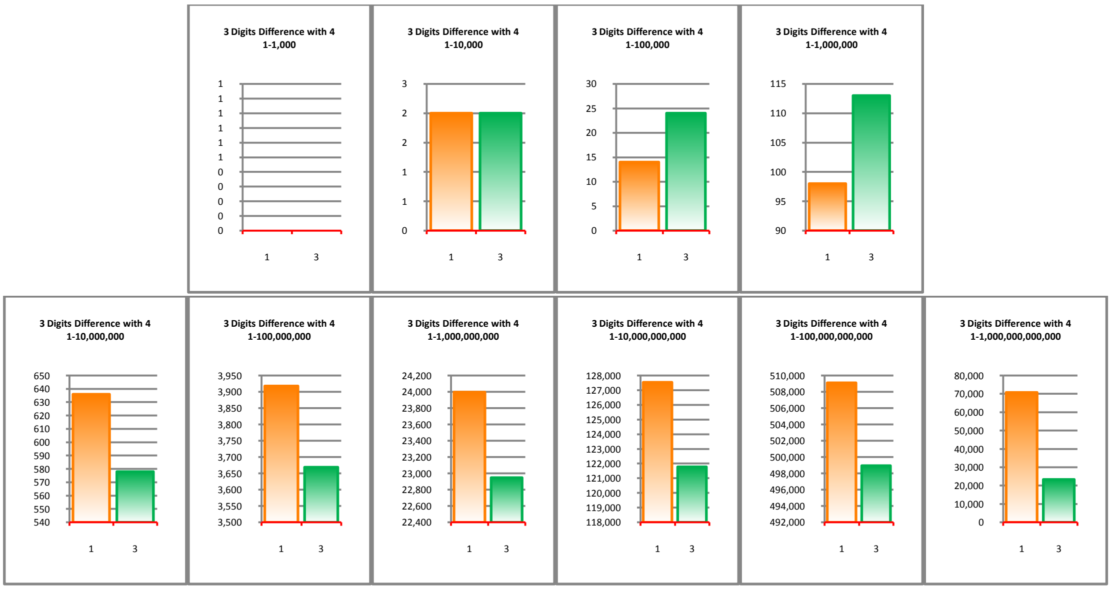



**Figure 7:** Differences of Number of Primes having ThreeSuccessive 1's and 3's in their Digits with those having ThreeSuccessive 4's in them in Ranges of $1 - 10^n$.

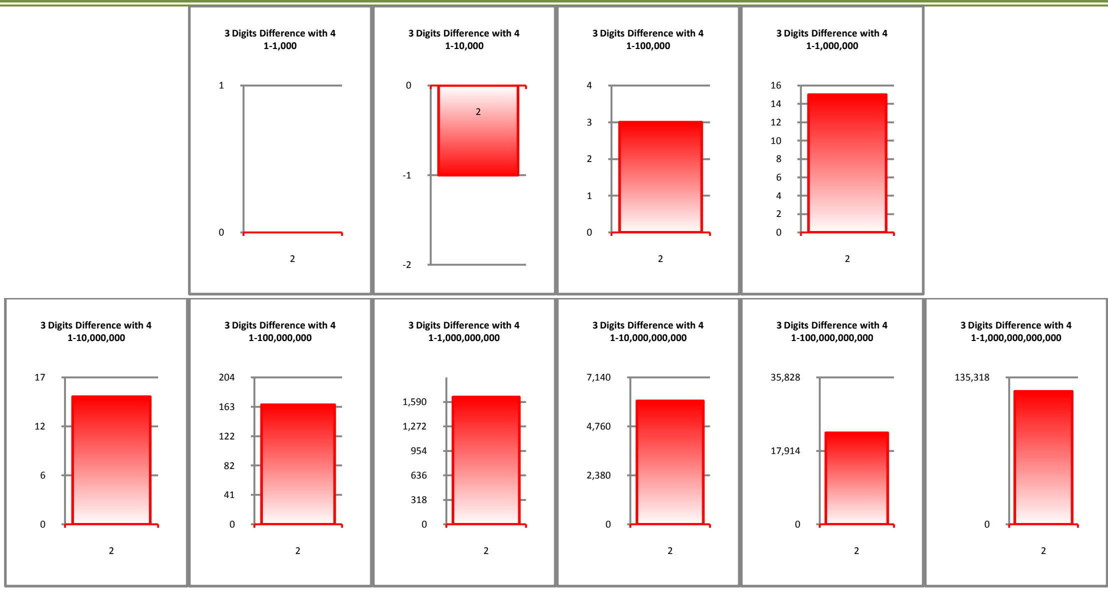



**Figure 8:** Difference of Number of Primes having ThreeSuccessive 2's in their Digits with those having ThreeSuccessive 4's in them in Ranges of 1 – $10^n$.

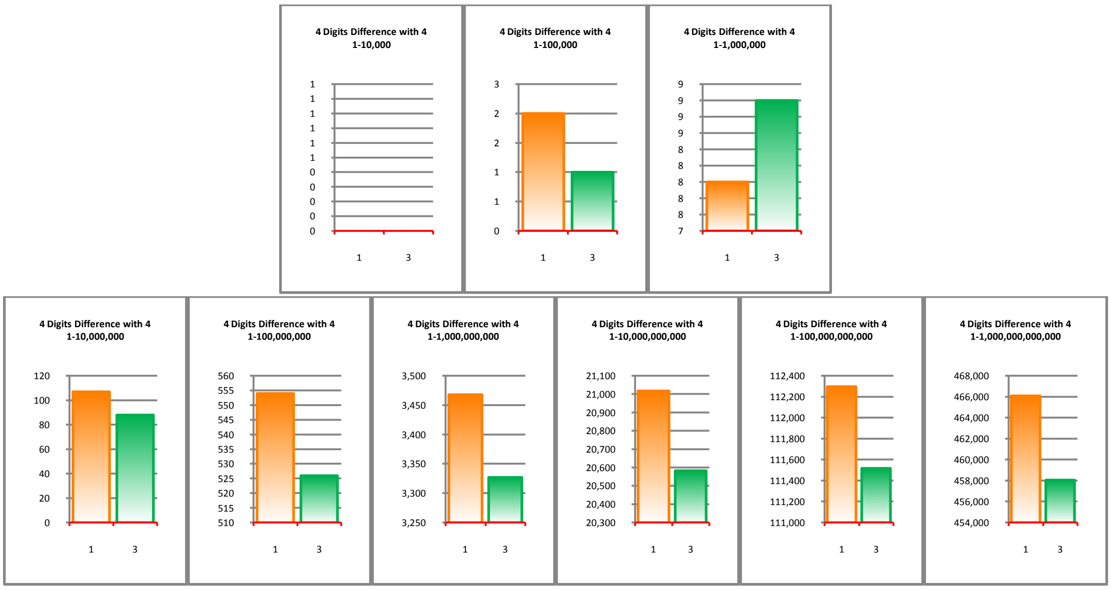
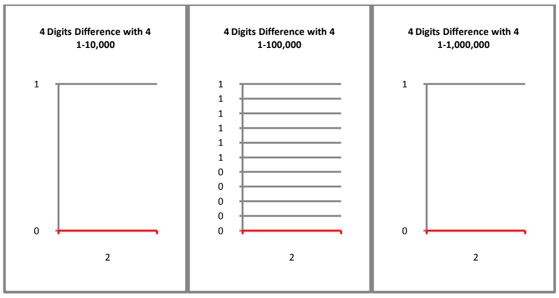



**Figure 9:** Differences of Number of Primes having FourSuccessive 1's and 3's in their Digits with those having FourSuccessive 4's in them in Ranges of 1 – $10^n$.



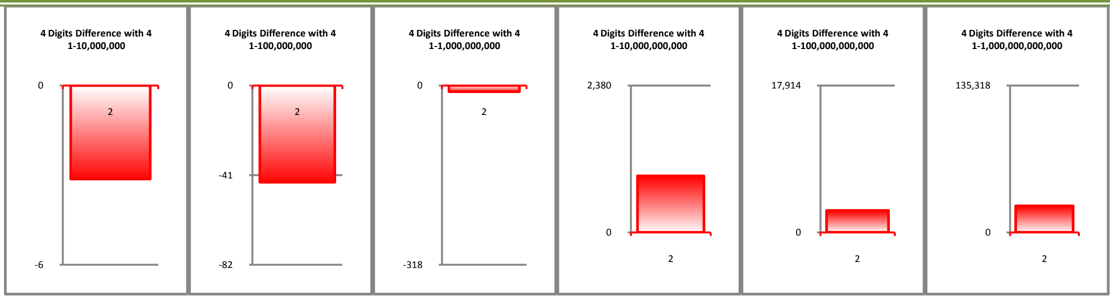



**Figure 10:** Difference of Number of Primes having FourSuccessive 2's in their Digits with those having FourSuccessive 4's in them in Ranges of $1 – 10^n$.

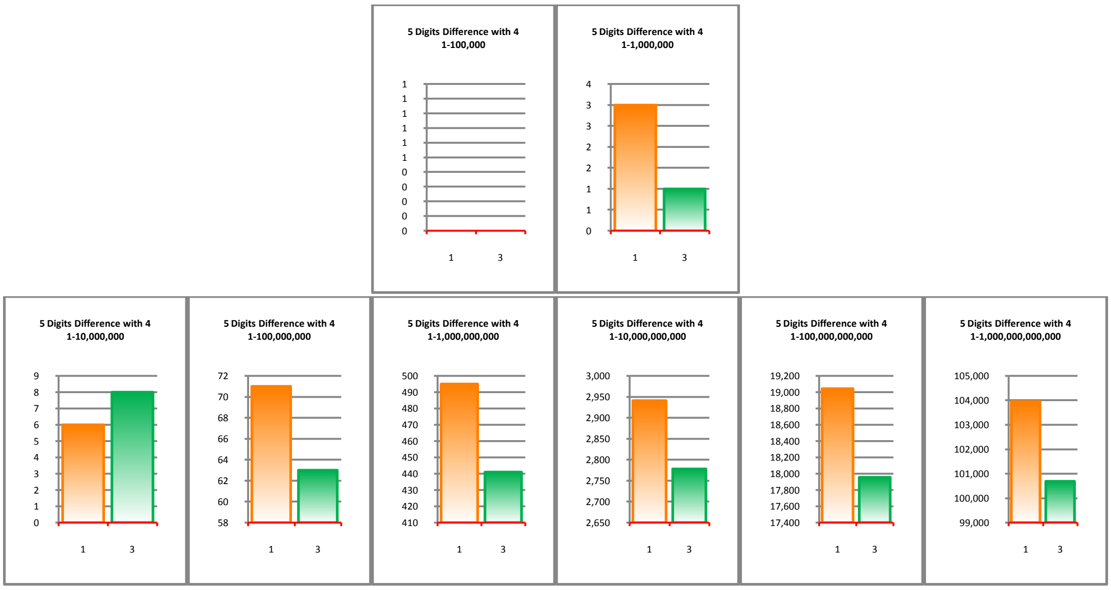



**Figure 11:** Differences of Number of Primes having FiveSuccessive 1's and 3's in their Digits with those having FiveSuccessive 4's in them in Ranges of $1 – 10^n$.

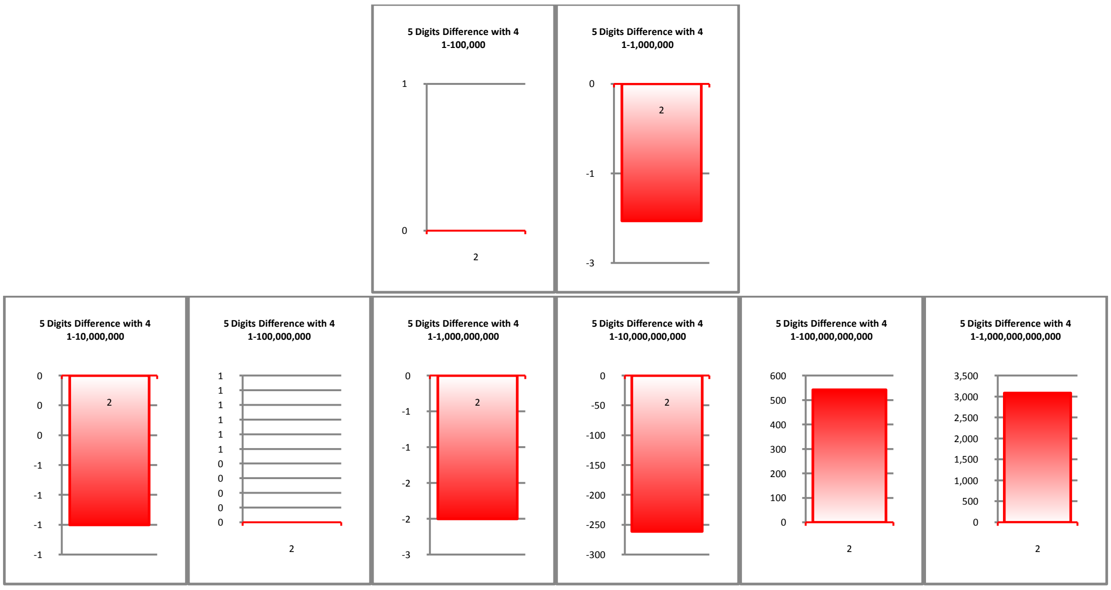



**Figure 12:** Difference of Number of Primes having FiveSuccessive 2's in their Digits with those having FiveSuccessive 4's in them in Ranges of $1 – 10^n$.

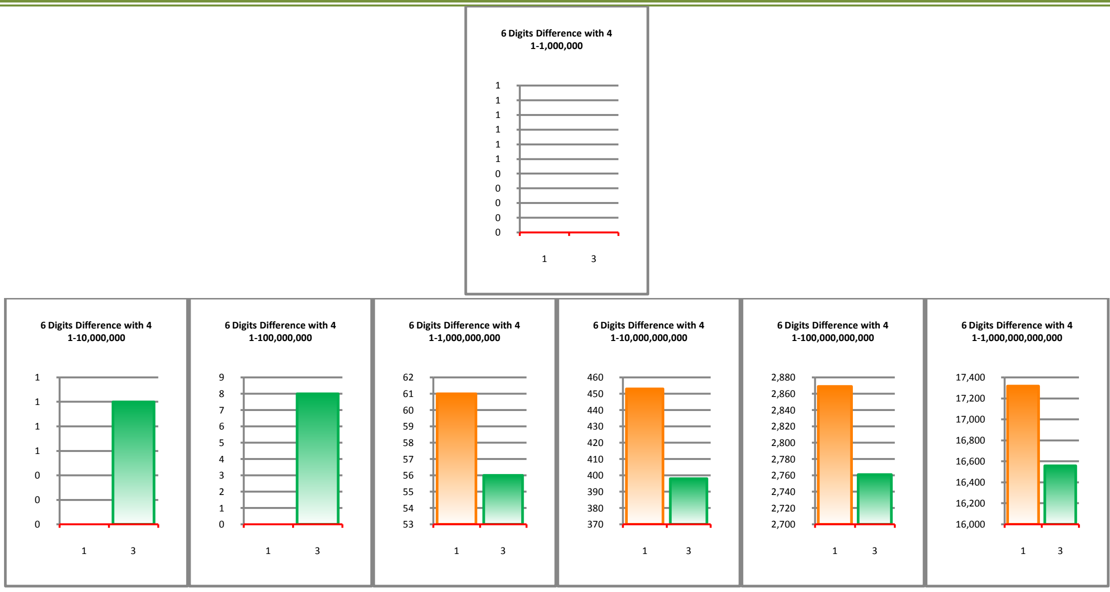



**Figure 13:** Differences of Number of Primes having SixSuccessive 1's and 3's in their Digits with those having SixSuccessive 4's in them in Ranges of $1 – 10^n$.

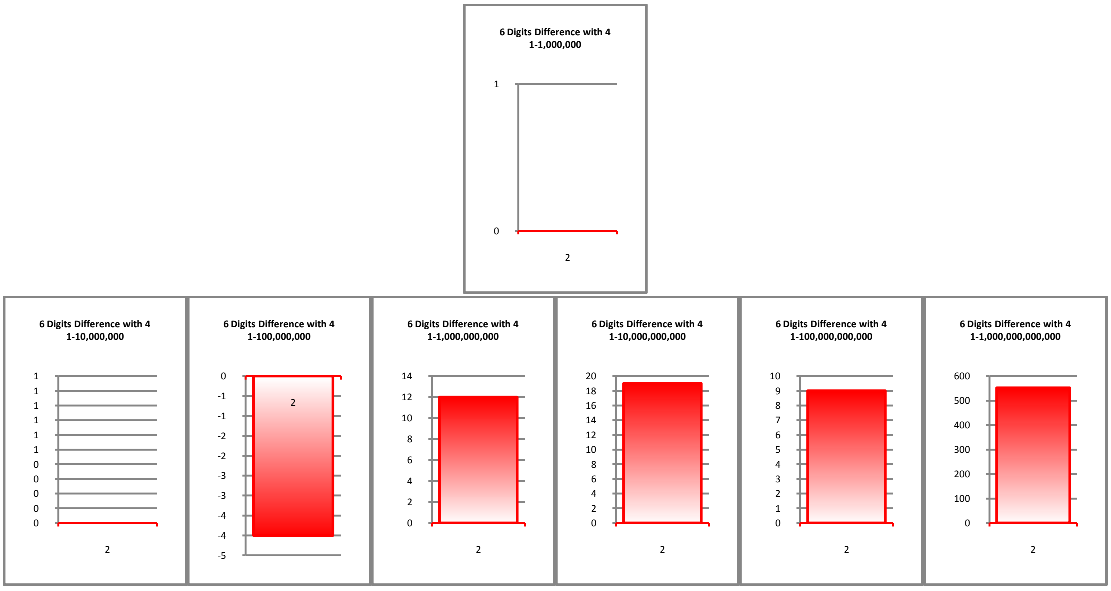



**Figure 14:** Difference of Number of Primes having SixSuccessive 2's in their Digits with those having SixSuccessive 4's in them in Ranges of $1 – 10^n$.

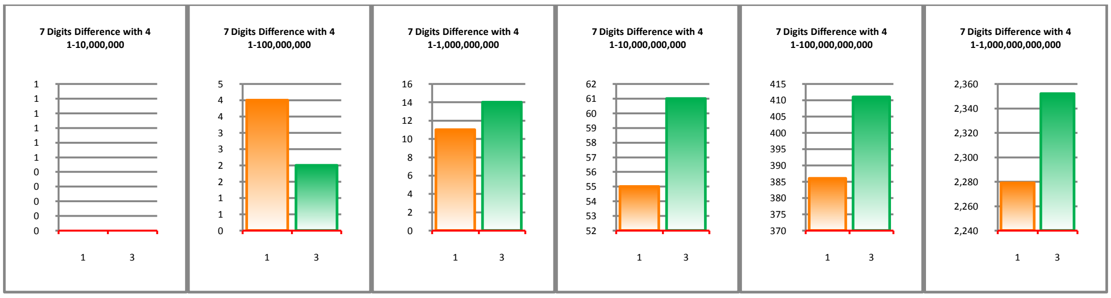



**Figure 15:** Differences of Number of Primes having SevenSuccessive 1's and 3's in their Digits with those having SevenSuccessive 4's in them in Ranges of $1 – 10^n$.

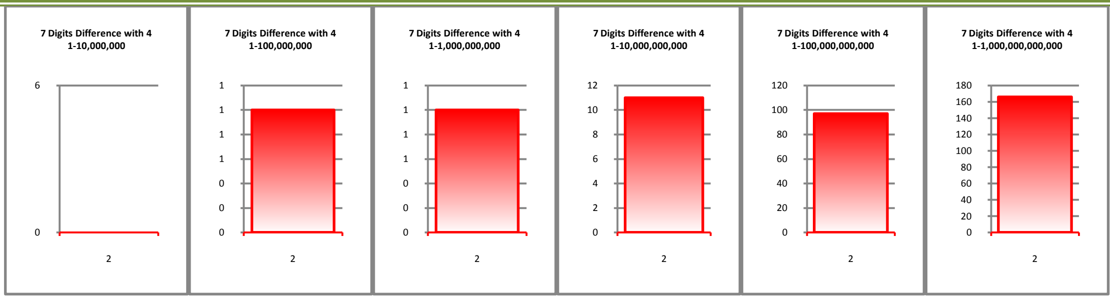



**Figure 16:** Difference of Number of Primes having SevenSuccessive 2's in their Digits with those having SevenSuccessive 4's in them in Ranges of 1 – $10^n$.

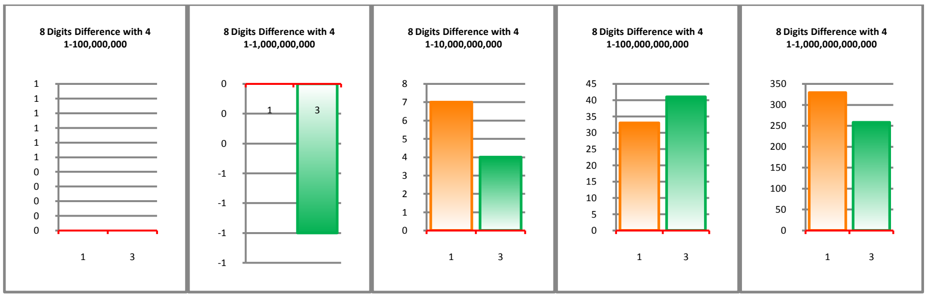



**Figure 17:** Differences of Number of Primes having EightSuccessive 1's and 3's in their Digits with those having EightSuccessive 4's in them in Ranges of 1 – $10^n$.

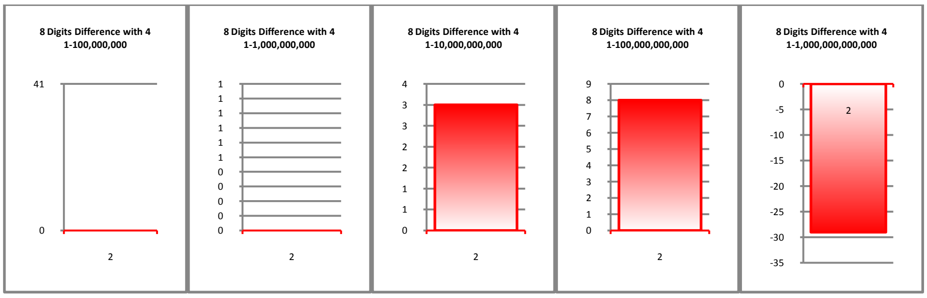



**Figure 18:** Difference of Number of Primes having EightSuccessive 2's in their Digits with those having EightSuccessive 4's in them in Ranges of 1 – $10^n$.

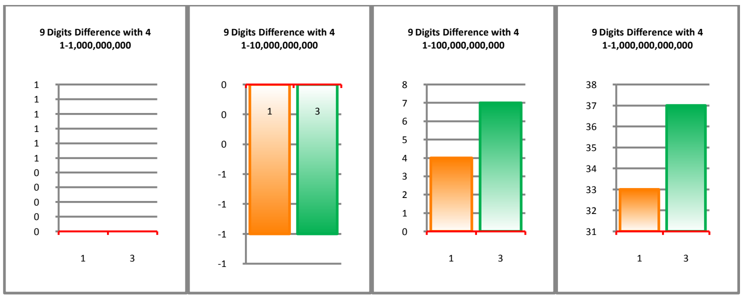



**Figure 19:** Differences of Number of Primes having NineSuccessive 1's and 3's in their Digits with those having NineSuccessive 4's in them in Ranges of 1 – $10^n$.

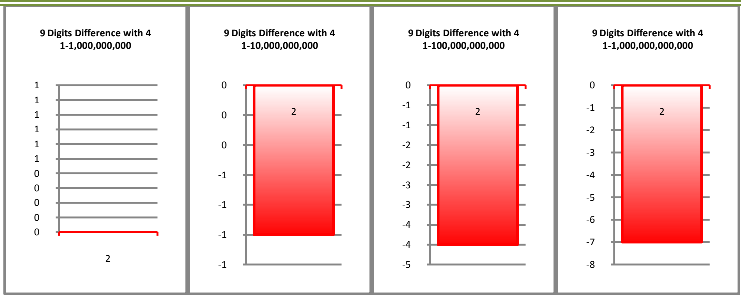



**Figure 20:** Difference of Number of Primes having NineSuccessive 2's in their Digits with those having NineSuccessive 4's in them in Ranges of $1 – 10^n$.

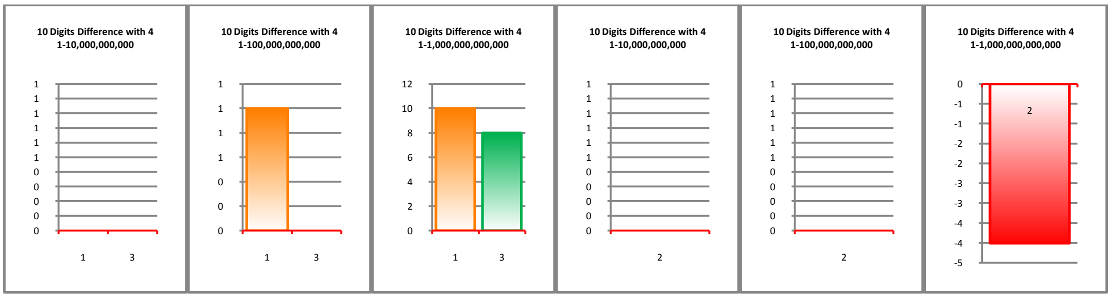



**Figure 21:** Differences of Number of Primes having TenSuccessive 1's&3'sand having Ten Successive 2's in their Digits with those having TenSuccessive 4's in them in Ranges of $1 – 10^n$.

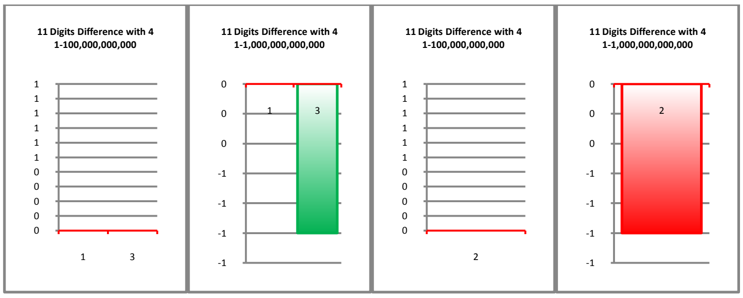



**Figure 22:** Differences of Number of Primes having ElevenSuccessive 1's&3'sand having Eleven Successive 2's in their Digits with those having ElevenSuccessive 4's in them in Ranges of $1 – 10^n$.

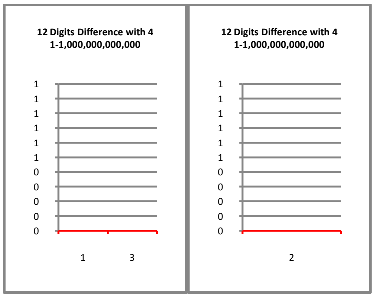



**Figure 23:** Differences of Number of Primes having TwelveSuccessive 1's&3'sand having Twelve Successive 2's in their Digits with those having TwelveSuccessive 4's in them in Ranges of $1 – 10^n$.

## IV. FIRST OCCURRENCE OF SUCCESSIVE DIGIT 4'S IN PRIME NUMBERS

The first positive integer containing 4 is 4 itself. For enough higher ranges, first occurrence of 2 4's is in 44, of 3 is in 444 and so on. The very first occurrence of multiple 4's is also of that of successive 4's.

**Formula 1 [12]** : If $n$ and $r$ are natural numbers, then the first occurrence of $r$ number of successive 4's in numbers in range $1 \le m < 10^n$ is

$$f = \begin{cases} - & \text{, if } r > n \\ \sum_{j=0}^{r-1}\left(4\times 10^{j}\right), & \text{if } r \le n \end{cases}.$$

Lack of such formula for successive occurrences of digits in primes makes their determination intensively computational.

**Table 3** :First Prime Numbers in Various Ranges with Multiple Successive 4's in Their Digits

| Sr. No. | Range | First Prime Number in Range with | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 Successive 4 | 2 Successive 4's | 3 Successive 4's | 4 Successive 4's | 5 Successive 4's | 6 Successive 4's |
| 1. | $1 - 10^1$ | - | - | - | - | - | - |
| 2. | $1 - 10^2$ | 41 | - | - | - | - | - |
| 3. | $1 - 10^3$ | 41 | 443 | - | - | - | - |
| 4. | $1 - 10^4$ | 41 | 443 | 4,441 | - | - | - |
| 5. | $1 - 10^5$ | 41 | 443 | 4,441 | 44,449 | - | - |
| 6. | $1 - 10^6$ | 41 | 443 | 4,441 | 44,449 | 444,443 | - |
| 7. | $1 - 10^7$ | 41 | 443 | 4,441 | 44,449 | 444,443 | - |
| 8. | $1 - 10^8$ | 41 | 443 | 4,441 | 44,449 | 444,443 | 24,444,443 |
| 9. | $1 - 10^9$ | 41 | 443 | 4,441 | 44,449 | 444,443 | 24,444,443 |
| 10. | $1 - 10^{10}$ | 41 | 443 | 4,441 | 44,449 | 444,443 | 24,444,443 |
| 11. | $1 - 10^{11}$ | 41 | 443 | 4,441 | 44,449 | 444,443 | 24,444,443 |
| 12. | $1 - 10^{12}$ | 41 | 443 | 4,441 | 44,449 | 444,443 | 24,444,443 |

**Table 3** :Continued …

| Sr. No. | Range | First Prime Number in Range with | | | | |
|---|---|---|---|---|---|---|
| | | 7 Successive 4's | 8Successive 4's | 9Successive 4's | 10Successive 4's | 11Successive 4's |
| 1. | $1 - 10^1$ | - | - | - | - | - |
| 2. | $1 - 10^2$ | - | - | - | - | - |
| 3. | $1 - 10^3$ | - | - | - | - | - |
| 4. | $1 - 10^4$ | - | - | - | - | - |
| 5. | $1 - 10^5$ | - | - | - | - | - |
| 6. | $1 - 10^6$ | - | - | - | - | - |
| 7. | $1 - 10^7$ | - | - | - | - | - |
| 8. | $1 - 10^8$ | - | - | - | - | - |
| 9. | $1 - 10^9$ | 444,444,421 | 444,444,443 | - | - | - |
| 10. | $1 - 10^{10}$ | 444,444,421 | 444,444,443 | 4,444,444,447 | - | - |
| 11. | $1 - 10^{11}$ | 444,444,421 | 444,444,443 | 4,444,444,447 | 44,444,444,441 | - |
| 12. | $1 - 10^{12}$ | 444,444,421 | 444,444,443 | 4,444,444,447 | 44,444,444,441 | 444,444,444,443 |

Quite often overall first occurrences of multiple 4's are successive only.

## V. LAST OCCURRENCE OF SUCCESSIVE DIGIT 4'S IN PRIME NUMBERS

Last natural number, in ranges $1 - 10^n$, containing $r$ number of successive non-zero digits fits in a systematic formula, which is obviously for 4 also.

**Formula 2 [12]** : If $n$ and $r$ are natural numbers, then the last occurrence of $r$ successive 4's in numbers in range $1 \le m < 10^n$ is

$$l = \begin{cases} - & \text{, if } r > n \\ \sum_{j=0}^{r-1}\left(4\times10^{j}\right) + \begin{cases} 0 & \text{, if } r = n \\ \sum_{j=r}^{n-1}\left(9\times10^{j}\right) & \text{, if } r < n \end{cases} \end{cases}.$$

Working on the prime numbers, we have determined the largest prime with $r$ number of successive 4's in them in these ranges.

**Table 4** :Last Prime Numbers in Various Ranges with Multiple Successive 4's in Their Digits

| Sr. No. | Number of Successive 4's | Last Prime Number in Range 1 – | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $10^1$ | $10^2$ | $10^3$ | $10^4$ | $10^5$ | $10^6$ | $10^7$ | $10^8$ |
| 1. | 1 | - | 47 | 947 | 9,949 | 99,643 | 999,749 | 9,999,943 | 99,999,941 |
| 2. | 2 | - | - | 449 | 8,447 | 98,443 | 998,443 | 9,998,447 | 99,998,449 |
| 3. | 3 | - | - | - | 4,447 | 94,447 | 994,447 | 9,994,441 | 99,984,449 |
| 4. | 4 | - | - | - | - | 44,449 | 844,447 | 9,944,449 | 99,944,447 |
| 5. | 5 | - | - | - | - | - | 444,449 | 7,444,441 | 98,444,443 |
| 6. | 6 | - | - | - | - | - | - | - | 74,444,449 |
| 7. | 7 | - | - | - | - | - | - | - | - |
| 8. | 8 | - | - | - | - | - | - | - | - |
| 9. | 9 | - | - | - | - | - | - | - | - |
| 10. | 10 | - | - | - | - | - | - | - | - |
| 11. | 11 | - | - | - | - | - | - | - | - |

**Table 4** :Continued …

| Sr. No. | Number of Successive 4's | Last Prime Number in Range 1 – | | |
|---|---|---|---|---|
| | | $10^9$ | $10^{10}$ | $10^{11}$ |
| 1. | 1 | 999,999,541 | 9,999,999,943 | 99,999,999,947 |
| 2. | 2 | 999,994,487 | 9,999,994,409 | 99,999,996,443 |
| 3. | 3 | 999,974,447 | 9,999,974,449 | 99,999,994,447 |
| 4. | 4 | 999,944,441 | 9,999,944,447 | 99,999,644,449 |
| 5. | 5 | 999,444,449 | 9,998,444,441 | 99,999,444,443 |
| 6. | 6 | 994,444,447 | 9,994,444,441 | 99,974,444,447 |
| 7. | 7 | 944,444,441 | 9,544,444,447 | 99,944,444,449 |
| 8. | 8 | 444,444,443 | 5,444,444,443 | 99,444,444,443 |
| 9. | 9 | - | 4,444,444,447 | 84,444,444,443 |
| 10. | 10 | - | - | 44,444,444,441 |
| 11. | 11 | - | - | - |

The remark for general occurrences of 4's applies equally to successive occurrences of 4's.

**Remark** : The maximum number of successive 4's in any prime number in the range 1 - $10^n$ is at most $n - 1$.

The numbers determined in all sections of this paper form new integer sequences and must be treated separately.

## Acknowledgements

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