# Low Density Distribution of 3-PrimeFactors Numbers till 1 Trillion

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**Abstract:** After generalizing prime numbers to 'k-PrimeFactors numbers', the first of its kind with non-trivial value 2 of k has been recently analyzed in detail. 3-PrimeFactors numbers also deserve such deep analysis and this work is the beginning of that saga. First number of 3-PrimeFactors numbers is increasing ranges are determined. There the smallest and largest such number is also noted. Then main focus is shifted to the minimality with determination of minimum count of these numbers in blocks of various sizes of powers of 10 till 1 trillion, first and last blocks containing minimum 3-PrimeFactors numbers, and total number of such blocks. Then keeping block-size fixed, increasing ranges are inspected for the study of the same kind for different block sizes.

**Keywords:** Prime number, k-PrimeFactors number, 3-PrimeFactors number, Low density distribution

**Mathematics Subject Classification (2010):** 11A51, 11N05, 11N80

## I. Introduction

Mathematics is science of systematic formulation. It begins technically with numbers. Amongst numbers, the simplest ones are positive integers, also called appropriately as natural numbers. Every natural number greater than 1 is uniquely represented as product of prime numbers [1]. So prime numbers have utmost importance in number theory. Another equally strong reason for primes enjoying status of importance is that they lack precise formulation. This forces study of primes either on approximate scale or by exhaustively drilling all integers within higher and higher ranges for hunt of primes [2] and analyzing all of them there [3]. Similar policy needs be adopted for special types of primes [4].

The author has identified classes of new numbers based on prime numbers [6].

**Definition** (*k*-PrimeFactors Number) : For any integer $k \geq 0$, a positive integer having $k$ number of prime factors, which need not be necessarily distinct, is called as *k*-PrimeFactors number.

Since primes are infinite in numbers, each type of *k*-PrimeFactors numbers is also infinite in number. As primes are randomly distributed, so are these *k*-PrimeFactors numbers also expected to be.

## II. 3-PrimeFactors Numbers

For particular value of $k$ as 3, we get 3-PrimeFactors numbers.

**Definition** (3-PrimeFactors Number) : A positive integer having exactly 3 prime divisors, not necessarily distinct, is called as 3-PrimeFactors number.

2-PrimeFactors numbers have been analyzed from different perspectives [6], [7], [8], [9], [10], [11].

First few 3-PrimeFactors numbers are :

$$8, 12, 18, 20, 28, 30, \cdots$$

where $8 = 2^3$, $12 = 2^2 \times 3$, $18 = 2 \times 3^2$, $20 = 2^2 \times 5$, $28 = 2^2 \times 7$, $30 = 2 \times 3 \times 5$ and so on.

## III. Number of 3-PrimeFactors Numbers till 1 Trillion

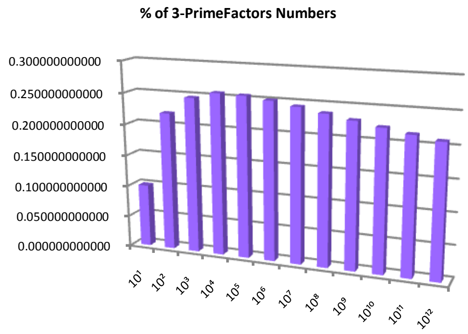
For the analysis of 3-PrimeFactors numbers done during this work, first all prime numbers in required ranges were determined and then their products led to the database of 3-PrimeFactors numbers. Java programming language [5] was chosen to run on modern electronic computers which made this work possible.

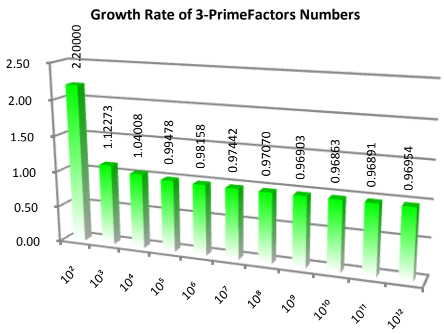
| *Sr. No.* | *Range* | *Number of 3-PrimeFactors Numbers in Range* | *First 3-PrimeFactors Number in Range* | *Last 3-PrimeFactors Number in Range* |
|---|---|---|---|---|
| 1 | $<10^1$ | 1 | 8 | 8 |
| 2 | $<10^2$ | 22 | 8 | 99 |
| 3 | $<10^3$ | 247 | 8 | 994 |
| 4 | $<10^4$ | 2,569 | 8 | 9,994 |

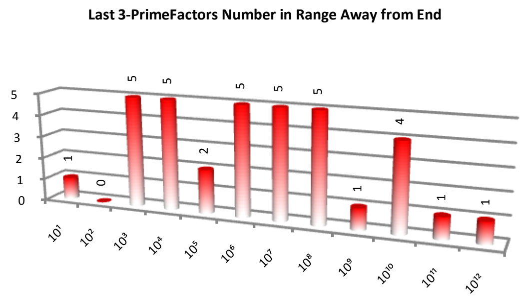
  

| *Sr. No.* | *Range* | *Number of 3-PrimeFactors Numbers in Range* | *First 3-PrimeFactors Number in Range* | *Last 3-PrimeFactors Number in Range* |
|---|---|---|---|---|
| 5 | <$10^5$ | 25,556 | 8 | 99,997 |
| 6 | <$10^6$ | 250,853 | 8 | 999,994 |
| 7 | <$10^7$ | 2,444,359 | 8 | 9,999,994 |
| 8 | <$10^8$ | 23,727,305 | 8 | 99,999,994 |
| 9 | <$10^9$ | 229,924,367 | 8 | 999,999,998 |
| 10 | <$10^{10}$ | 2,227,121,996 | 8 | 9,999,999,995 |
| 11 | <$10^{11}$ | 21,578,747,909 | 8 | 99,999,999,998 |
| 12 | <$10^{12}$ | 209,214,982,911 | 8 | 999,999,999,998 |

Obviously number of 3-PrimeFactors numbers increases with increase in the inspection range. But their percentage is seen lowering. For all ranges, the first 3-PrimeFactors number is 8. Within various ranges, the last 3-PrimeFactors numbers remain quite near range-end.



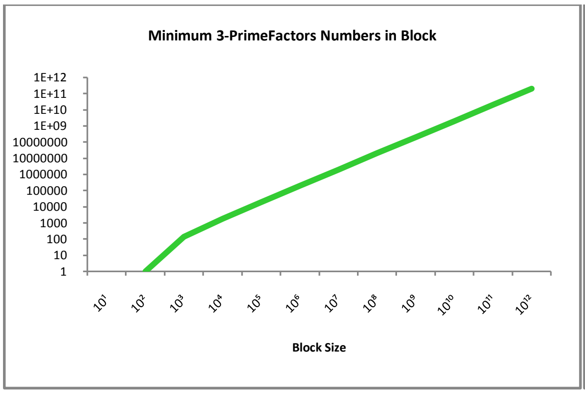

## IV. Minimum Number of 3-PrimeFactors Numbers in Blocks of Sizes $10^n$

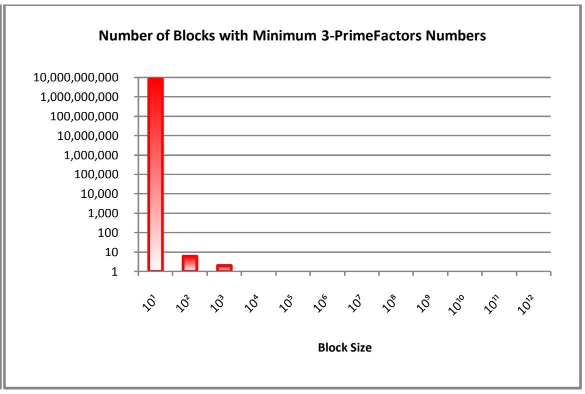
Whole range of numbers less than 1 trillion was covered and by choosing blocks of sizes of 10 powers like, 10, $10^2$, $10^3$ and so on till the trillion itself, the minimum number of 3-PrimeFactors numbers in occurring in these blocks, first and last blocks with such minimum 3-PrimeFactors numbers in them and number of blocks containing minimum numbers are found to be as follows. The very first block is represented by 0, second by 1 × BlockSize and so on.

| *Sr. No.* | *Block-Size* | *Minimum 3-PrimeFactors Numbers in Block* | *First Block of Minimum 3-PrimeFactors Numbers* | *Last Block of Minimum 3-PrimeFactors Numbers* | *No. of Blocks with Minimum 3-PrimeFactors Numbers* |
|---|---|---|---|---|---|
| 1 | $10^1$ | 0 | 80 | 999,999,999,980 | 8,302,155,790 |
| 2 | $10^2$ | 1 | 516,628,703,800 | 988,235,695,200 | 6 |
| 3 | $10^3$ | 143 | 647,491,209,000 | 906,595,627,000 | 2 |
| 4 | $10^4$ | 1,876 | 873,427,890,000 | 873,427,890,000 | 1 |
| 5 | $10^5$ | 20,140 | 991,516,300,000 | 991,516,300,000 | 1 |

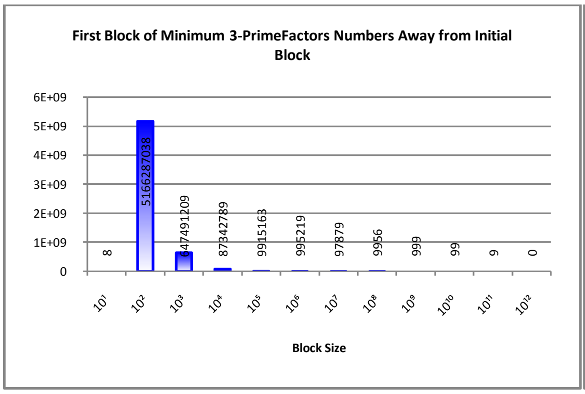
| *Sr. No.* | *Block-Size* | *Minimum 3-PrimeFactors Numbers in Block* | *First Block of Minimum 3-PrimeFactors Numbers* | *Last Block of Minimum 3-PrimeFactors Numbers* | *No. of Blocks with Minimum 3-PrimeFactors Numbers* |
|---|---|---|---|---|---|
| 6 | $10^6$ | 205,154 | 995,219,000,000 | 995,219,000,000 | 1 |
| 7 | $10^7$ | 2,061,834 | 978,790,000,000 | 978,790,000,000 | 1 |
| 8 | $10^8$ | 20,639,030 | 995,600,000,000 | 995,600,000,000 | 1 |
| 9 | $10^9$ | 206,437,397 | 999,000,000,000 | 999,000,000,000 | 1 |
| 10 | $10^{10}$ | 2,064,543,373 | 990,000,000,000 | 990,000,000,000 | 1 |
| 11 | $10^{11}$ | 20,658,070,805 | 900,000,000,000 | 900,000,000,000 | 1 |
| 12 | $10^{12}$ | 209,214,982,911 | 0 | 0 | 1 |

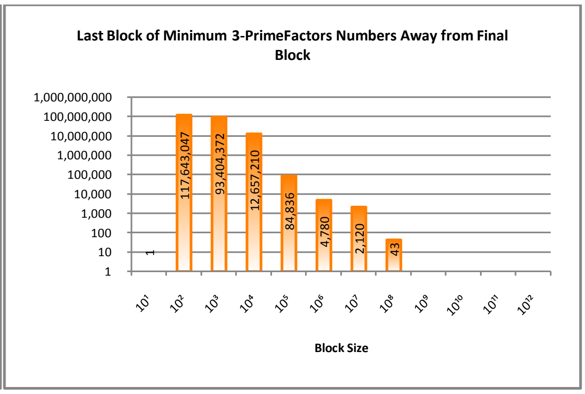
The minimum number of 3-PrimeFactors numbers within blocks of increasing size goes on increasing with higher percentage of occurrence. But there are fewer blocks of larger size containing respective minimum 3-PrimeFactors numbers in them.




The first and last blocks of various sizes containing minimum number of 3-PrimeFactors numbers are following block distances away for initial and final blocks respectively.



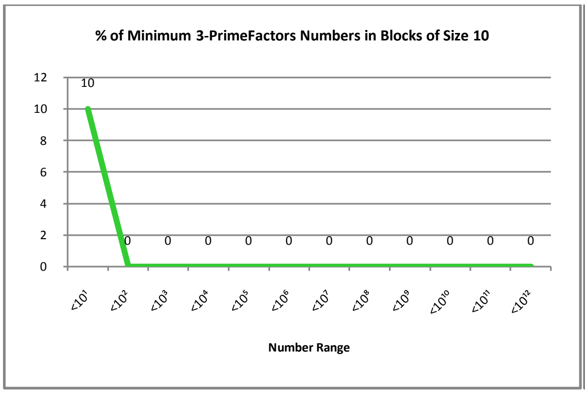

After this analysis of 3-PrimeFactors numbers in complete range of 1 trillion for different block sizes; for block of each size, similar analysis in increasing ranges is undertaken.

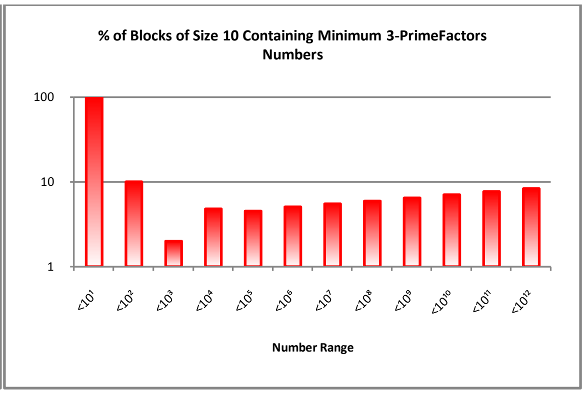
### IV.1. Minimum Number of 3-PrimeFactors Numbers in Blocks of Size 10

We start with block size 10 for searching minimum number of 3-PrimeFactors numbers in block, first and last block of minimum numbers and number of such blocks. For this size, block 0 means first block 0 to 9, block 10 means second block from 10 to 19 and so on.

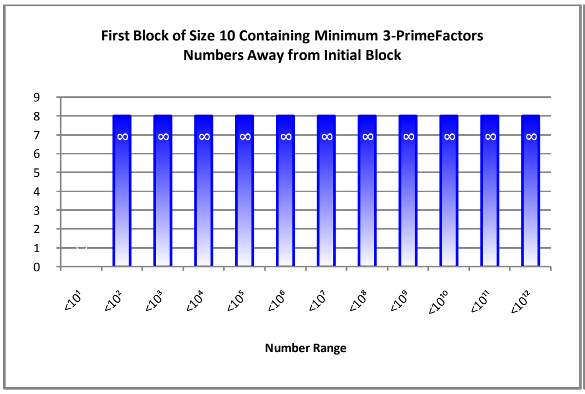
| Sr. No. | Range | Minimum 3-PrimeFactors Numbers in 10-Size Block | First 10-Size Block of Minimum 3-PrimeFactors Numbers | Last 10-Size Block of Minimum 3-PrimeFactors Numbers | Number of 10-Size Blocks with Minimum 3-PrimeFactors Numbers |
|---|---|---|---|---|---|
| 1 | $<10^1$ | 1 | 0 | 0 | 1 |
| 2 | $<10^2$ | 0 | 80 | 80 | 1 |
| 3 | $<10^3$ | 0 | 80 | 300 | 2 |
| 4 | $<10^4$ | 0 | 80 | 9,930 | 48 |
| 5 | $<10^5$ | 0 | 80 | 99,980 | 453 |
| 6 | $<10^6$ | 0 | 80 | 999,490 | 5,069 |
| 7 | $<10^7$ | 0 | 80 | 9,999,290 | 55,033 |
| 8 | $<10^8$ | 0 | 80 | 99,999,840 | 594,259 |
| 9 | $<10^9$ | 0 | 80 | 999,999,740 | 6,462,922 |
| 10 | $<10^{10}$ | 0 | 80 | 9,999,999,950 | 70,377,372 |
| 11 | $<10^{11}$ | 0 | 80 | 99,999,999,880 | 765,492,029 |
| 12 | $<10^{12}$ | 0 | 80 | 999,999,999,980 | 8,302,155,790 |

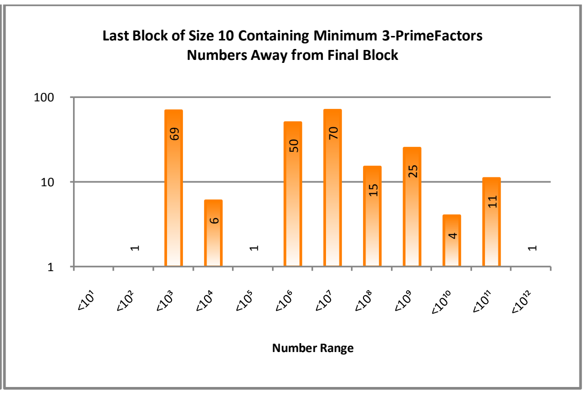
Except for first range, the minimum 3-PrimeFactors numbers in blocks of size 10 for higher ranges is 0. There is variation in the percentage of number of blocks of size 10 containing minimum 3-PrimeFactors numbers for different ranges.




Starter block of minimality is at a fixed block distance 8 from initial block and last such block has varying block distance with final blocks.



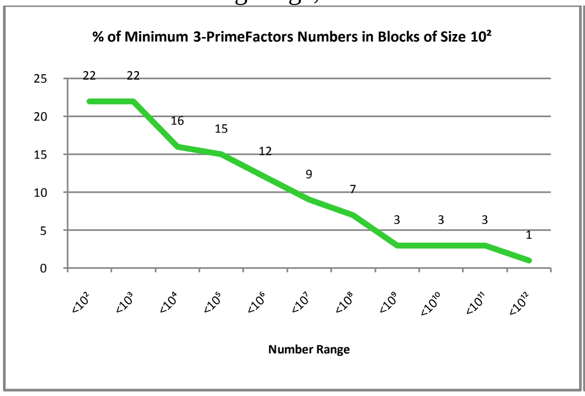

**IV.2. Minimum Number of 3-PrimeFactors Numbers in Blocks of Size $10^2$**

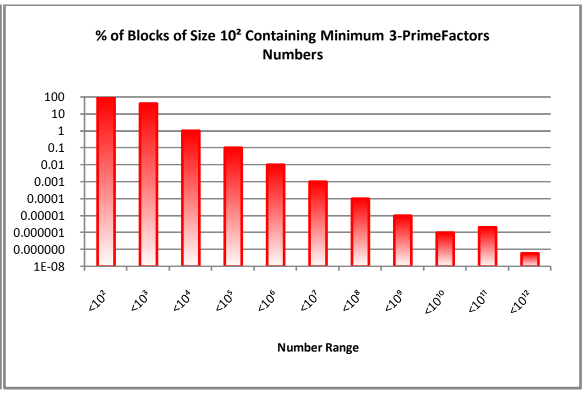
Next block of study is of size $10^2$, i.e., 100. So, here block 0 stands for number range 0 to 99, block 100 stands for number range 100 to 199 and so on.

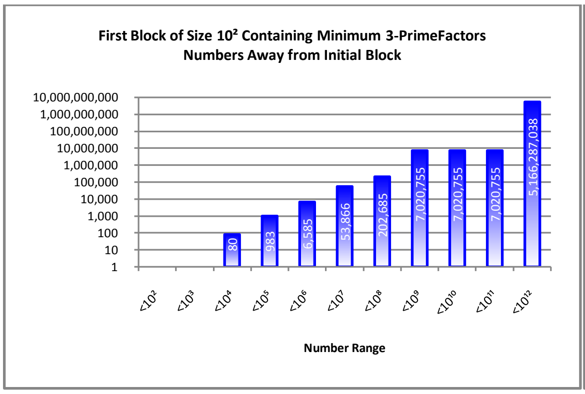
| *Sr. No.* | *Range* | *Minimum 3-PrimeFactors Numbers in $10^2$-Size Block* | *First $10^2$-Size Block of Minimum 3-PrimeFactors Numbers* | *Last $10^2$-Size Block of Minimum 3-PrimeFactors Numbers* | *Number of $10^2$-Size Blocks with Minimum 3-PrimeFactors Numbers* |
|---|---|---|---|---|---|
| 1 | $<10^2$ | 22 | 0 | 0 | 1 |
| 2 | $<10^3$ | 22 | 0 | 800 | 4 |
| 3 | $<10^4$ | 16 | 8,000 | 8,000 | 1 |
| 4 | $<10^5$ | 15 | 98,300 | 98,300 | 1 |
| 5 | $<10^6$ | 12 | 658,500 | 658,500 | 1 |
| 6 | $<10^7$ | 9 | 5,386,600 | 5,386,600 | 1 |
| 7 | $<10^8$ | 7 | 20,268,500 | 20,268,500 | 1 |
| 8 | $<10^9$ | 3 | 702,075,500 | 702,075,500 | 1 |
| 9 | $<10^{10}$ | 3 | 702,075,500 | 702,075,500 | 1 |
| 10 | $<10^{11}$ | 3 | 702,075,500 | 96,498,363,700 | 21 |
| 11 | $<10^{12}$ | 1 | 516,628,703,800 | 988,235,695,200 | 6 |

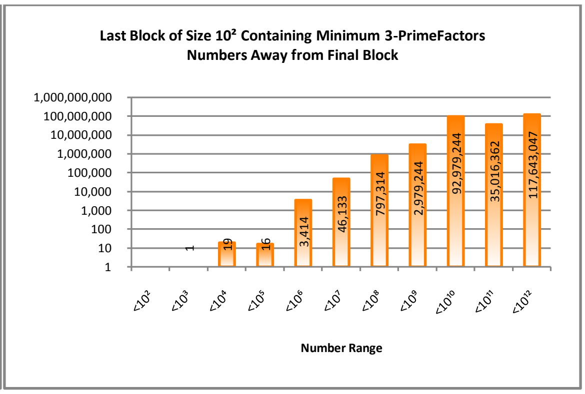
With increasing range, number of minimum 3-PrimeFactors numbers in blocks of size 100 decreases.




The first and last blocks of minimum 3-PrimeFactors numbers stay more and more away from respective ends.



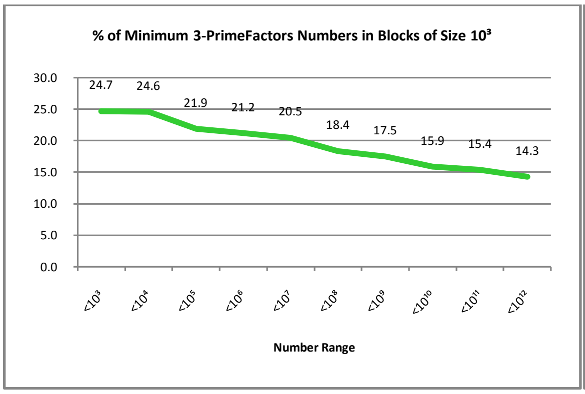

**IV.3. Minimum Number of 3-PrimeFactors Numbers in Blocks of Size $10^3$**

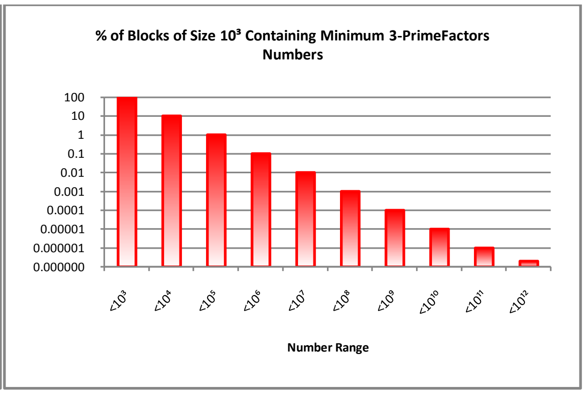
$10^3$, i.e., 1000, is size of block now. Here block 0 represents number range 0 to 999, block 1000 represents number range 1000 to 1999 and so on.

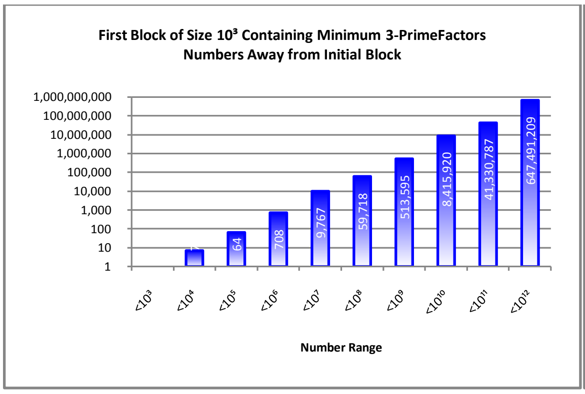
| *Sr. No.* | *Range* | *Minimum 3-PrimeFactors Numbers in $10^3$-Size Block* | *First $10^3$-Size Block of Minimum 3-PrimeFactors Numbers* | *Last $10^3$-Size Block of Minimum 3-PrimeFactors Numbers* | *Number of $10^3$-Size Blocks with Minimum 3-PrimeFactors Numbers* |
|---|---|---|---|---|---|
| 1 | $<10^3$ | 247 | 0 | 0 | 1 |

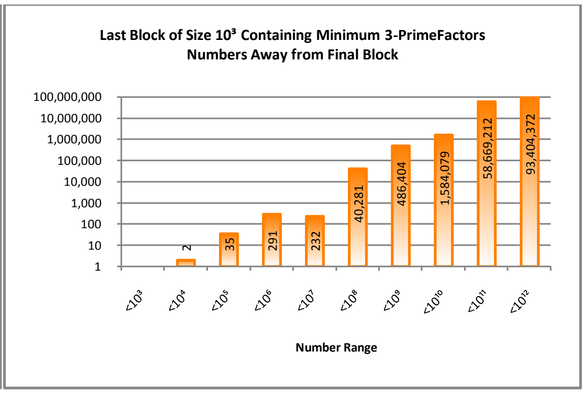
| Sr. No. | Range | Minimum 3-PrimeFactors Numbers in $10^3$-Size Block | First $10^3$-Size Block of Minimum 3-PrimeFactors Numbers | Last $10^3$-Size Block of Minimum 3-PrimeFactors Numbers | Number of $10^3$-Size Blocks with Minimum 3-PrimeFactors Numbers |
|---|---|---|---|---|---|
| 2 | $<10^4$ | 246 | 7,000 | 7,000 | 1 |
| 3 | $<10^5$ | 219 | 64,000 | 64,000 | 1 |
| 4 | $<10^6$ | 212 | 708,000 | 708,000 | 1 |
| 5 | $<10^7$ | 205 | 9,767,000 | 9,767,000 | 1 |
| 6 | $<10^8$ | 184 | 59,718,000 | 59,718,000 | 1 |
| 7 | $<10^9$ | 175 | 513,595,000 | 513,595,000 | 1 |
| 8 | $<10^{10}$ | 159 | 8,415,920,000 | 8,415,920,000 | 1 |
| 9 | $<10^{11}$ | 154 | 41,330,787,000 | 41,330,787,000 | 1 |
| 10 | $<10^{12}$ | 143 | 647,491,209,000 | 906,595,627,000 | 2 |

The values are analysed graphically.




### IV.4. Minimum Number of 3-PrimeFactors Numbers in Blocks of Size $10^4$

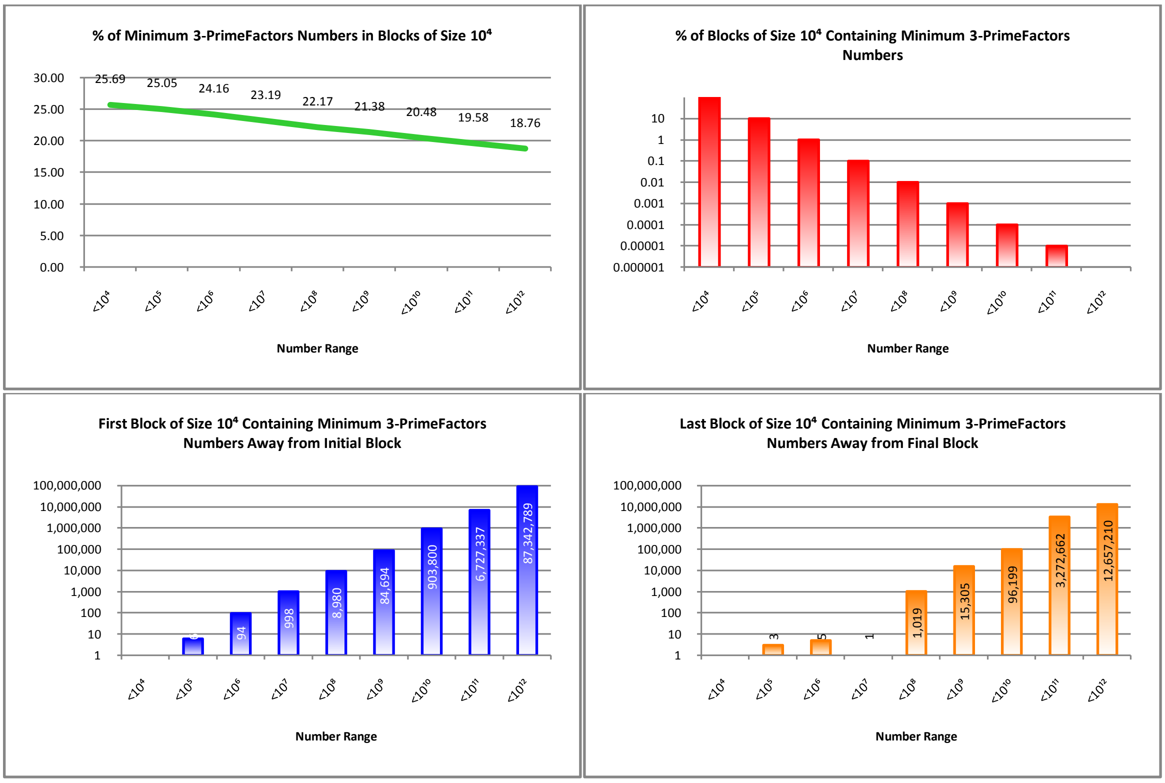
Now its turn of block size $10^4$, i.e., 10000, block 0 indicating number range 0 to 9999, block 10000 indicating number range 10000 to 19999 and so on.

| Sr. No. | Range | Minimum 3-PrimeFactors Numbers in $10^4$-Size Block | First $10^4$-Size Block of Minimum 3-PrimeFactors Numbers | Last $10^4$-Size Block of Minimum 3-PrimeFactors Numbers | Number of $10^4$-Size Blocks with Minimum 3-PrimeFactors Numbers |
|---|---|---|---|---|---|
| 1 | $<10^4$ | 2,569 | 0 | 0 | 1 |
| 2 | $<10^5$ | 2,505 | 60,000 | 60,000 | 1 |
| 3 | $<10^6$ | 2,416 | 940,000 | 940,000 | 1 |
| 4 | $<10^7$ | 2,319 | 9,980,000 | 9,980,000 | 1 |
| 5 | $<10^8$ | 2,217 | 89,800,000 | 89,800,000 | 1 |

| Sr. No. | Range | Minimum 3-PrimeFactors Numbers in $10^4$-Size Block | First $10^4$-Size Block of Minimum 3-PrimeFactors Numbers | Last $10^4$-Size Block of Minimum 3-PrimeFactors Numbers | Number of $10^4$-Size Blocks with Minimum 3-PrimeFactors Numbers |
|---|---|---|---|---|---|
| 6 | $<10^9$ | 2,138 | 846,940,000 | 846,940,000 | 1 |
| 7 | $<10^{10}$ | 2,048 | 9,038,000,000 | 9,038,000,000 | 1 |
| 8 | $<10^{11}$ | 1,958 | 67,273,370,000 | 67,273,370,000 | 1 |
| 9 | $<10^{12}$ | 1,876 | 873,427,890,000 | 873,427,890,000 | 1 |

For higher ranges, the minimality continues to decrease and first container block goes on farther from initial block.

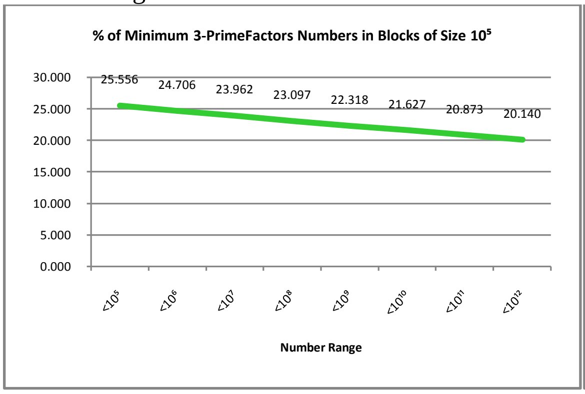


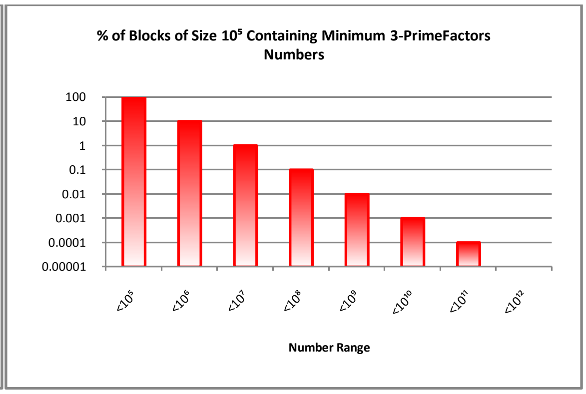

## IV.5. Minimum Number of 3-PrimeFactors Numbers in Blocks of Size $10^5$

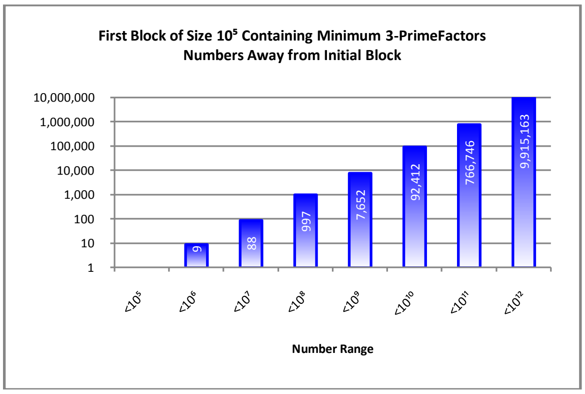
After $10^4$, its block size $10^5$, i.e., 100000, under inspection; wherein block 0 gives number range 0 to 99999, block 100000 gives number range 100000 to 199999 and so on.

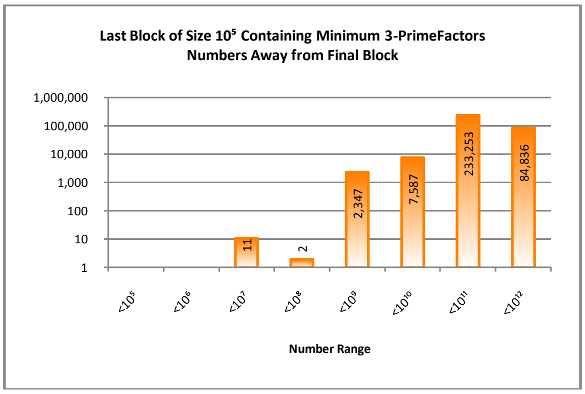
| Sr. No. | Range | Minimum 3-PrimeFactors Numbers in $10^5$-Size Block | First $10^5$-Size Block of Minimum 3-PrimeFactors Numbers | Last $10^5$-Size Block of Minimum 3-PrimeFactors Numbers | Number of $10^5$-Size Blocks with Minimum 3-PrimeFactors Numbers |
|---|---|---|---|---|---|
| 1 | $<10^5$ | 25,556 | 0 | 0 | 1 |
| 2 | $<10^6$ | 24,706 | 900,000 | 900,000 | 1 |
| 3 | $<10^7$ | 23,962 | 8,800,000 | 8,800,000 | 1 |
| 4 | $<10^8$ | 23,097 | 99,700,000 | 99,700,000 | 1 |
| 5 | $<10^9$ | 22,318 | 765,200,000 | 765,200,000 | 1 |
| 6 | $<10^{10}$ | 21,627 | 9,241,200,000 | 9,241,200,000 | 1 |
| 7 | $<10^{11}$ | 20,873 | 76,674,600,000 | 76,674,600,000 | 1 |
| 8 | $<10^{12}$ | 20,140 | 991,516,300,000 | 991,516,300,000 | 1 |

In this case also, the occasions of lesser frequency of occurrences of 3-PrimeFactors numbers do come with increasing effect.



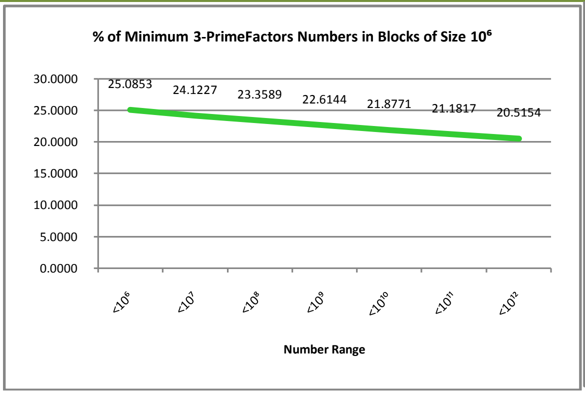



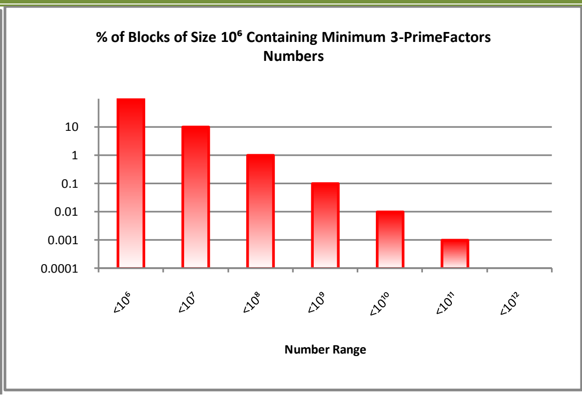

### IV.6. Minimum Number of 3-PrimeFactors Numbers in Blocks of Size $10^6$

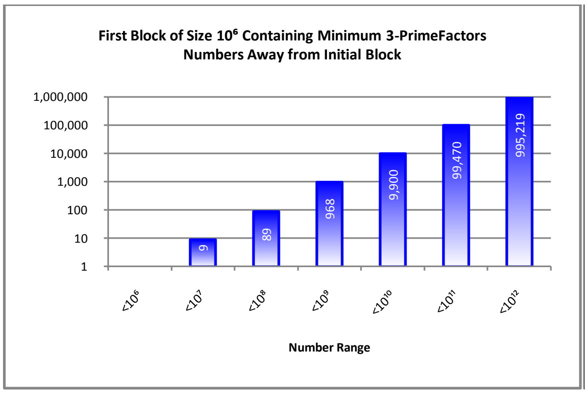
Now its turn of block size $10^6$, i.e., 1000000, block 0 being number range 0 to 999999, block 1000000 being number range 1000000 to 1999999 and so on.

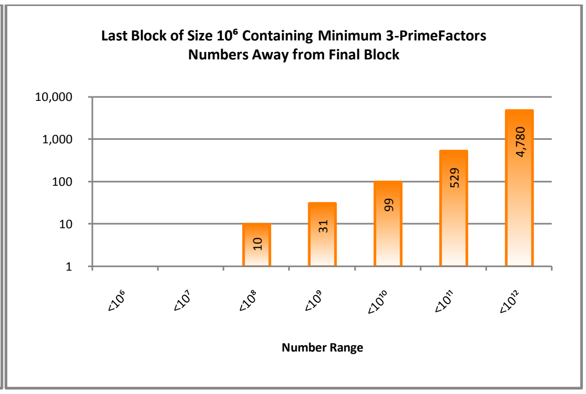
| *Sr. No.* | *Range* | *Minimum 3-PrimeFactors Numbers in $10^6$-Size Block* | *First $10^6$-Size Block of Minimum 3-PrimeFactors Numbers* | *Last $10^6$-Size Block of Minimum 3-PrimeFactors Numbers* | *Number of $10^6$-Size Blocks with Minimum 3-PrimeFactors Numbers* |
|---|---|---|---|---|---|
| 1 | $<10^6$ | 250,853 | 0 | 0 | 1 |
| 2 | $<10^7$ | 241,227 | 9,000,000 | 9,000,000 | 1 |
| 3 | $<10^8$ | 233,589 | 89,000,000 | 89,000,000 | 1 |
| 4 | $<10^9$ | 226,144 | 968,000,000 | 968,000,000 | 1 |
| 5 | $<10^{10}$ | 218,771 | 9,900,000,000 | 9,900,000,000 | 1 |
| 6 | $<10^{11}$ | 211,817 | 99,470,000,000 | 99,470,000,000 | 1 |
| 7 | $<10^{12}$ | 205,154 | 995,219,000,000 | 995,219,000,000 | 1 |

Mostly blocks with lower density of 3-PrimeFactors numbers in them are situated quite near last end.



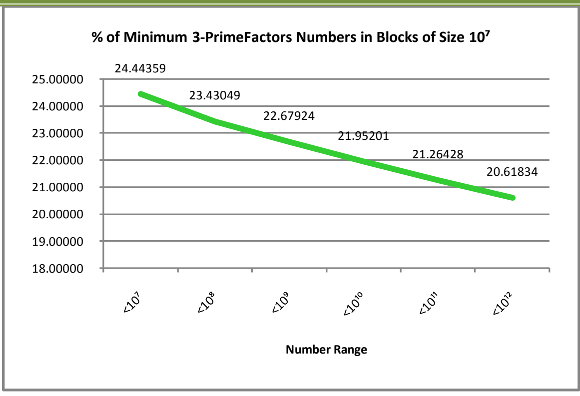

### IV.7. Minimum Number of 3-PrimeFactors Numbers in Blocks of Size $10^7$

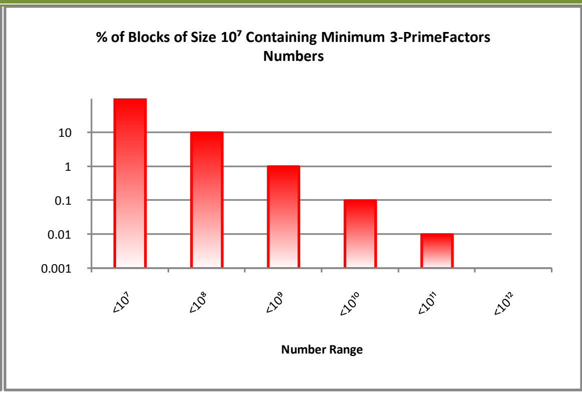
Higher block size is $10^7$, i.e., 10000000; block 0 just means number range 0 to 9999999, block 10000000 means number range 10000000 to 19999999 and so on.

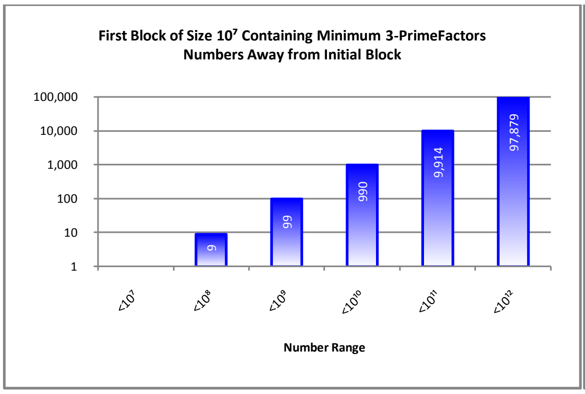
| *Sr. No.* | *Range* | *Minimum 3-PrimeFactors Numbers in $10^7$-Size Block* | *First $10^7$-Size Block of Minimum 3-PrimeFactors Numbers* | *Last $10^7$-Size Block of Minimum 3-PrimeFactors Numbers* | *Number of $10^7$-Size Blocks with Minimum 3-PrimeFactors Numbers* |
|---|---|---|---|---|---|
| 1 | $<10^7$ | 2,444,359 | 0 | 0 | 1 |
| 2 | $<10^8$ | 2,343,049 | 90,000,000 | 90,000,000 | 1 |
| 3 | $<10^9$ | 2,267,924 | 990,000,000 | 990,000,000 | 1 |
| 4 | $<10^{10}$ | 2,195,201 | 9,900,000,000 | 9,900,000,000 | 1 |
| 5 | $<10^{11}$ | 2,126,428 | 99,140,000,000 | 99,140,000,000 | 1 |
| 6 | $<10^{12}$ | 2,061,834 | 978,790,000,000 | 978,790,000,000 | 1 |

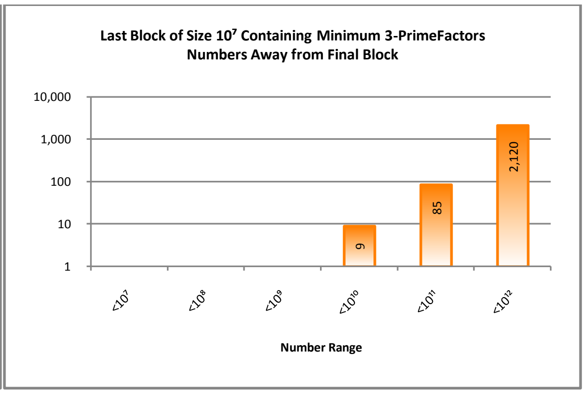
There are unique blocks of size $10^7$ with lowest density of 3-PrimeFactors numbers for all ranges under consideration.



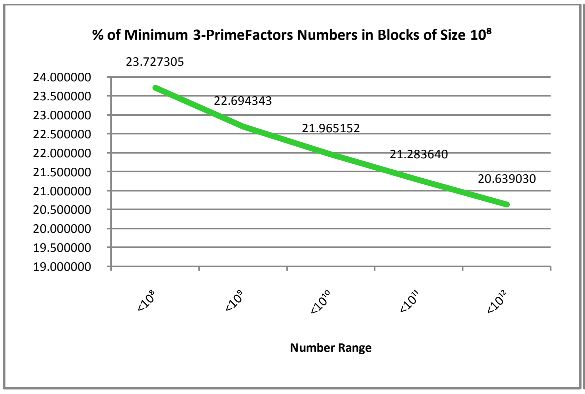



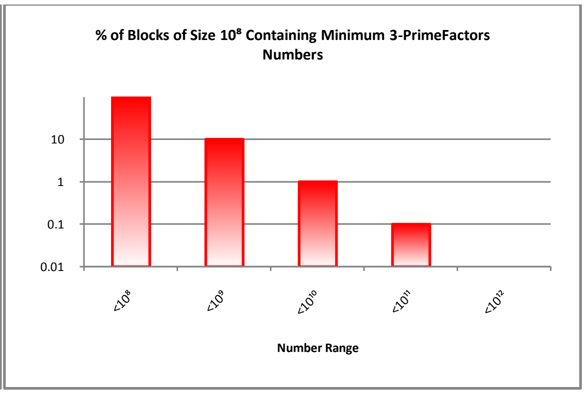

**IV.8. Minimum Number of 3-PrimeFactors Numbers in Blocks of Size $10^8$**

Next number is of block size $10^8$, i.e., 100000000. Here block 0 refers to range 0 to 99999999, block 100000000 to range 100000000 to 199999999 and so on.

| Sr. No. | Range | Minimum 3-PrimeFactors Numbers in $10^8$-Size Block | First $10^8$-Size Block of Minimum 3-PrimeFactors Numbers | Last $10^8$-Size Block of Minimum 3-PrimeFactors Numbers | Number of $10^8$-Size Blocks with Minimum 3-PrimeFactors Numbers |
|---|---|---|---|---|---|
| 1 | $<10^8$ | 23,727,305 | 0 | 0 | 1 |
| 2 | $<10^9$ | 22,694,343 | 900,000,000 | 900,000,000 | 1 |
| 3 | $<10^{10}$ | 21,965,152 | 9,900,000,000 | 9,900,000,000 | 1 |
| 4 | $<10^{11}$ | 21,283,640 | 99,600,000,000 | 99,600,000,000 | 1 |
| 5 | $<10^{12}$ | 20,639,030 | 995,600,000,000 | 995,600,000,000 | 1 |

Trends continue with minute variations.



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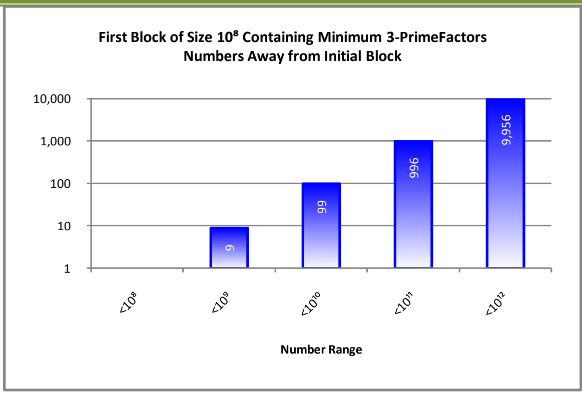
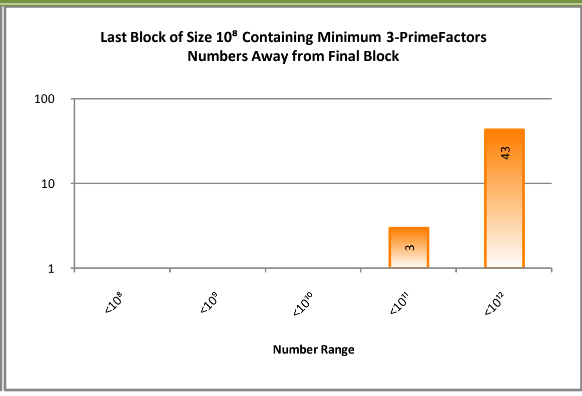


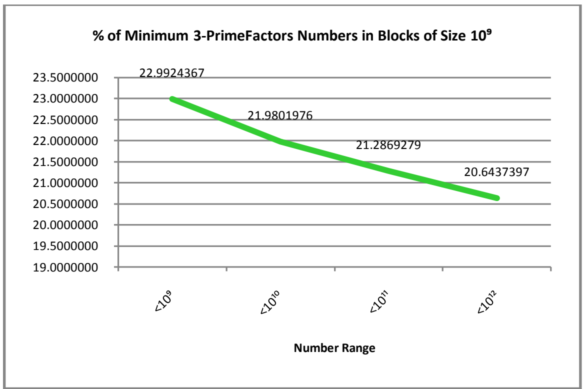



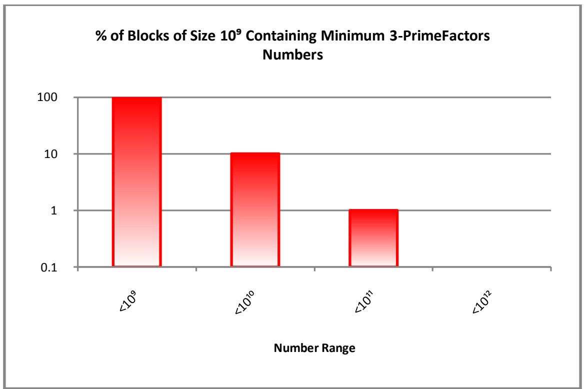

### IV.9. Minimum Number of 3-PrimeFactors Numbers in Blocks of Size $10^9$

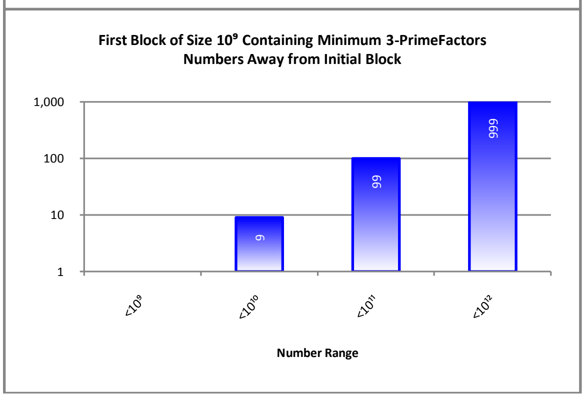
Next pick up is block size of $10^9$, i.e., 1000000000., block 0 corresponds to range 0 to 999999999, block 1000000000 corresponds to number range 1000000000 to 1999999999 and so on.

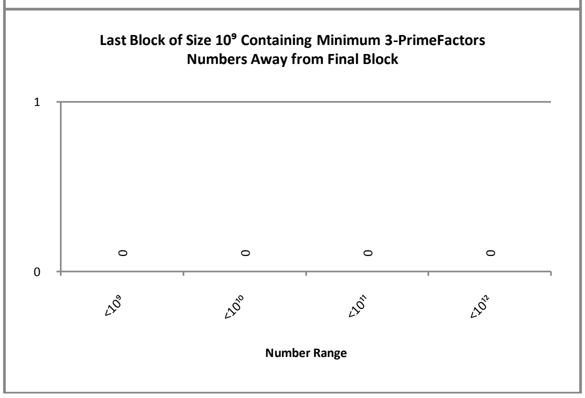
| *Sr. No.* | *Range* | *Minimum 3-PrimeFactors Numbers in $10^9$-Size Block* | *First $10^9$-Size Block of Minimum 3-PrimeFactors Numbers* | *Last $10^9$-Size Block of Minimum 3-PrimeFactors Numbers* | *Number of $10^9$-Size Blocks with Minimum 3-PrimeFactors Numbers* |
|---|---|---|---|---|---|
| 1 | $<10^9$ | 229,924,367 | 0 | 0 | 1 |
| 2 | $<10^{10}$ | 219,801,976 | 9,000,000,000 | 9,000,000,000 | 1 |
| 3 | $<10^{11}$ | 212,869,279 | 99,000,000,000 | 99,000,000,000 | 1 |
| 4 | $<10^{12}$ | 206,437,397 | 999,000,000,000 | 999,000,000,000 | 1 |

Lowest Density blocks continue to remain stuck to the ends.



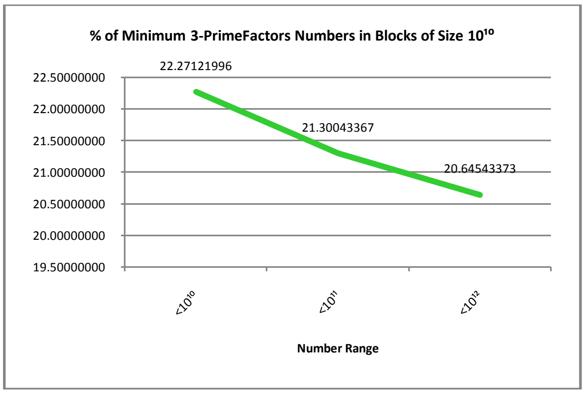



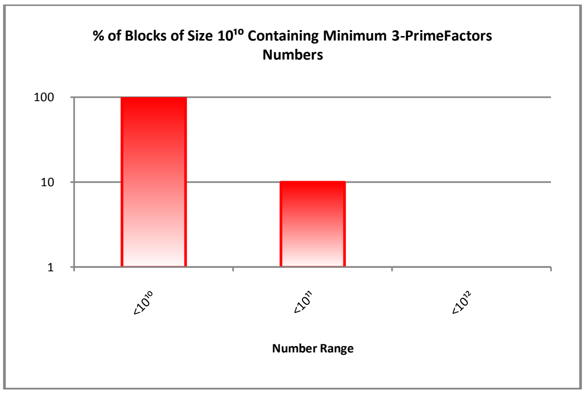
### IV.10. Minimum Number of 3-PrimeFactors Numbers in Blocks of Size $10^{10}$

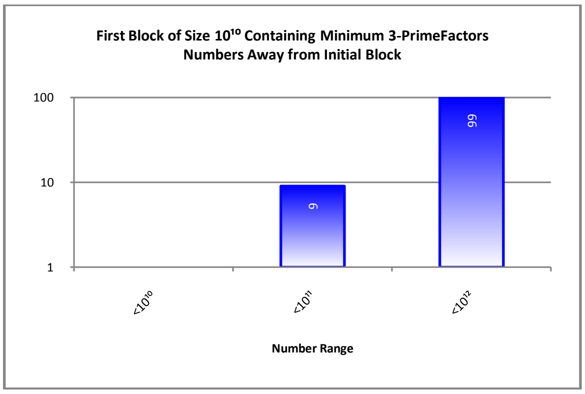
Further we take up block size $10^{10}$, i.e., 10000000000, block 0 denotes range 0 to 9999999999, block 10000000000 denotes number range 10000000000 to 19999999999 and so on.

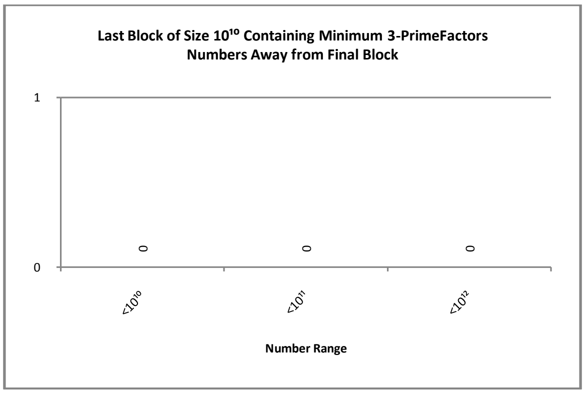
| *Sr. No.* | *Range* | *Minimum 3-PrimeFactors Numbers in $10^{10}$-Size Block* | *First $10^{10}$-Size Block of Minimum 3-PrimeFactors Numbers* | *Last $10^{10}$-Size Block of Minimum 3-PrimeFactors Numbers* | *Number of $10^{10}$-Size Blocks with Minimum 3-PrimeFactors Numbers* |
|---|---|---|---|---|---|
| 1 | $<10^{10}$ | 2,227,121,996 | 0 | 0 | 1 |
| 2 | $<10^{11}$ | 2,130,043,367 | 90,000,000,000 | 90,000,000,000 | 1 |
| 3 | $<10^{12}$ | 2,064,543,373 | 990,000,000,000 | 990,000,000,000 | 1 |

The number of blocks containing minimum number of 3-PrimeFactors numbers is also minimum, i.e., 1, within our range.



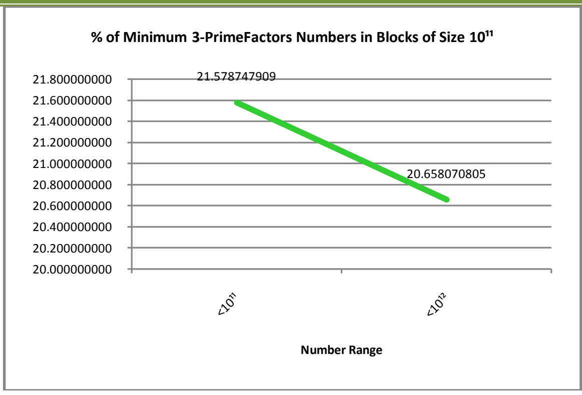

### IV.11. Minimum Number of 3-PrimeFactors Numbers in Blocks of Size $10^{11}$

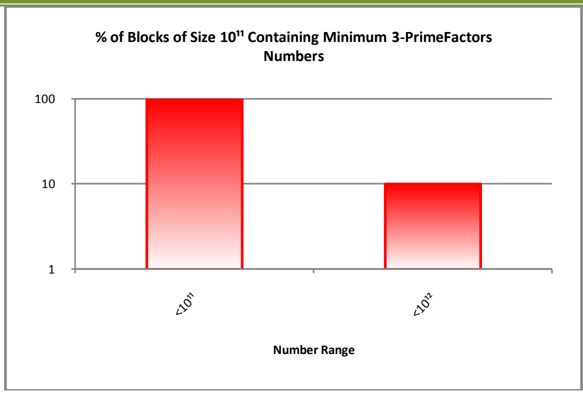
Higher block size is of $10^{11}$, i.e., 100000000000, where block 0 signifies number range 0 to 99999999999, block 100000000000 signifies number range 100000000000 to 199999999999 and so on.

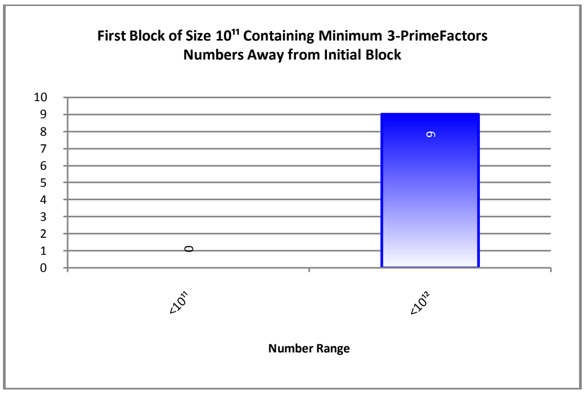
| *Sr. No.* | *Range* | *Minimum 3-PrimeFactors Numbers in $10^{11}$-Size Block* | *First $10^{11}$-Size Block of Minimum 3-PrimeFactors Numbers* | *Last $10^{11}$-Size Block of Minimum 3-PrimeFactors Numbers* | *Number of $10^{11}$-Size Blocks with Minimum 3-PrimeFactors Numbers* |
|---|---|---|---|---|---|
| 1 | $<10^{11}$ | 21,578,747,909 | 0 | 0 | 1 |
| 2 | $<10^{12}$ | 20,658,070,805 | 900,000,000,000 | 900,000,000,000 | 1 |

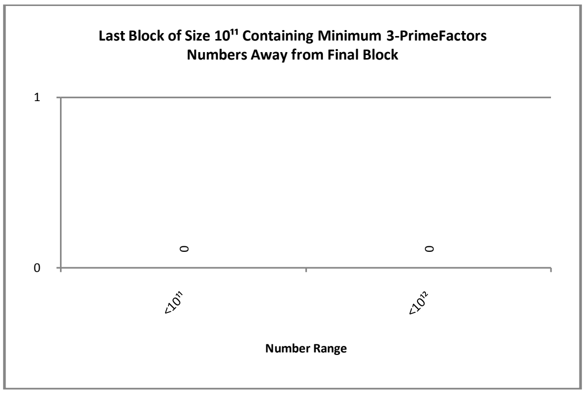
In this case also, the trends of the graphs are in harmony with those for earlier block sizes.




### IV.12. Minimum Number of 3-PrimeFactors Numbers in Blocks of Size $10^{12}$

The last block size of 1 trillion resembles with the whole study range itself. So the complete range itself is the only block of its size. It naturally is the first as well as the last block of minimum number of 3-PrimeFactors numbers in it. There are 209214982911 3-PrimeFactors numbers in it.

This analysis showed that 3-PrimeFactors numbers become rare only in higher ranges for blocks of all sizes. Also there happen to be lesser blocks with lowest densities of 3-PrimeFactors numbers.

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