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Properties of a mixture of bosons and fermions at very low temperature

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Abstract: Properties of a mixture of bosons and fermions at low temperature have been studied when the mixture is enclosed in a trap. Gross-Pitaevskii mean field equation for the boson distribution in the trap is solved by utilizing Thomas-Fermi Approximation to extract the density profile of the fermion and boson components. The results show that the Fermi gas will constitute a core enclosed by the Bose condensate when the boson-fermion interaction strength(h) is less than the boson-boson interaction strength(g) i.e. $h < g$. For $h = g$, the fermions have a constant spatial density where the bosons are localized and thus both condensates co-exist simultaneously. For $h > g$, fermions constitute a shell around a core of Bose condensate.

Key Words: Boson-Fermion mixtures, Boson-Boson interaction, Boson-Fermion interaction, Scattering lengths, Trapping potential..

1. Introduction

Since the experimental realization of Bose- Einstein condensation in dilute gases of Rubidium [1–4], Sodium [5, 6], Lithium [7], and Hydrogen [8]a great deal of interest in Bose Condensed systems have concentrated on the topic of multi- component condensates. This field was stimulated by the successful demonstration of overlapping condensates in different spin states of Rubidium in a magnetic trap [9, 10] and of Sodium in an optical trap [11], the (binary) mixtures being produced either by sympathetic cooling, which involves one species being cooled to below the transition temperature only through thermal contact with an already condensed Bose gas, or by radiative transitions out of a single component condensate. Since then, a host of experiments has been conducted on systems with two condensates, exploring both the dynamics of component separation [12], and measuring the relative quantum phase of the two Bose- Einstein condensates [13]. Most of the theoretical work concerning multi-component condensates [14 –23] has been devoted to systems of two Bose condensates. However, other systems are of fundamental interest. One of these being a Bose condensate with fermionic impurities, for instance a system ^{40}K -^{87}Rb, a Boson- Fermion mixture. In particular the possibility of sympathetic cooling of fermionic isotopes has been predicted in both ^{6}Li-^{7}Li [24], ^{39}K-^{40}K, and ^{41}K-^{40}K [25]. Magneto - optical trapping of the fermionic Potassium isotope ^{40}K has also been reported [26].

Quantum degeneracy was first reached with mixtures of bosonic $^{7}_{3}Li$ and fermionic $^{6}_{3}Li$[12], and also with $^{23}_{11}Na$ (bosonic) and$^{6}_{3}Li$ (fermionic) as well as ^{87}Rb (bosonic) and ^{40}K (fermionic) at ultra-low temperatures. These boson-fermion mixtures offer unique possibilities to study the effects of quantum statistics directly. Superfluidity has also been obtained experimentally in a mixture of Bose condensed gas and superfluid Fermi gas of two Lithium atoms, ^{6}Li and ^{7}Li [27], where a new mechanism of superfluidity and its instability was observed. In a Bose-Fermi superfluid mixture, there could be a fully mixed phase and a fully separated phase, and a third phase could consist of pure fermions in equilibrium with a mixture of bosons and fermions [28] or it could be pure bosons in equilibrium with a mixture of bosons and fermions. Ultimately the conditions for stability of homogenous phase of the mixture have to be studied. The phase diagram of a weakly interacting Boson-Fermion mixture at zero temperature can be derived starting from the following expression for energy density $E(n_F; n_B)$, i.e,

$$E(n_F; n_B) = \frac{1}{2} g_{BB} n_B^2 + g_{BF} n_B n_F + \frac{3}{5} E_F n_F \quad (1)$$

Where n_B is the density of bosons, n_F is the density of fermions and $E_F = \frac{\hbar^2 k_F^2}{2m_F}$, where k_F is the fermi momentum and m_F is the mass of the fermi particles; g_{BB} is the boson-boson inter-species coupling constant and g_{BF} is the boson-fermion coupling constant. These coupling constants are related to corresponding scattering lengths a_{BB} and a_{BF}. Recent experiments have shown that Boson-Fermion scattering length does not depend on the internal state of the Fermi atoms [29]. The stability condition predicted by the energy density of Eq. (1) for the uniform mixture is

$$n_F^{1/3} = \frac{\hbar^2 g_{BB}}{2m_F g_{BF}^2} \quad (2)$$

For n_F larger than this critical value, the uniform mixture is unstable and the system exhibits either partial or full phase separation. However, interacting superfluid fermions and the Bose-condensed bosons are coupled via the interspecies interaction term determined by g_{BF}. In general, the number of bosons and the number of fermions are not equal in the mixture. But the stability conditions of the mixture will depend on the relative number of bosons and fermions in the mixture, and this will also determine the phase separation, i.e., whether the box is filled with pure fermions, while bosons are still in the mixed phase in the remaining volume; and this corresponds to the partially mixed phase of the mixture.

Recently, bosonic lasers have been developed based on BEC of exciton-polaritons in semiconductor micro cavities. These electrically neutral bosons coexist with charged electrons and holes, which are thus Boson-Fermion mixtures. In the presence of magnetic fields, the charged particles are bound to their cyclotron orbits, while the neutral exciton-polaritons move freely. In this way the magnetic fields dramatically affect the phase diagram of a mixed Boson-Fermion mixture, switching between fermionic lasing, incoherent emission and bosonic lasing [30]

The trapping of the boson-fermion mixture is via the Feshbach resonance method. This is a method in which the spin dependence of the inter-atomic interaction gives rise to both open and closed channels. In the context of ultra-cold gases, they are of special importance as they allow the modification of the interactions between the atoms, in particular the scattering length [31]. A good example of such a mixture is ^{6}Li-^{40}K and ^{87}Rb-^{40}K [32]. More details about collisions among the atoms of gases can be found in [33]. It is also known that when the ^{87}Rb (boson) atoms are not completely evaporated, various regimes of mixtures are accessible, ranging from dense thermal ^{87}Rb cloud of 10^7, ^{87}Rb right at the phase transition point interacting with a moderately degenerate Fermi gas (^{40}K) of 2×10^6 atoms to deeply degenerate mixtures with almost pure condensate [34]. Stability conditions for Boson-Fermion mixtures have been studied leading to the values of N_B and N_F for stability [25].

There is also a recent experimental realization of Boson-Fermion superfluid mixtures of dilute ultra-cold atomic gases. Depending upon the values of the scattering lengths, and the amount of bosons and fermions, a uniform Boson-Fermion mixture could exhibit a fully mixed phase, or a fully separated phase, or a pure fermionic phase co-existing with a mixed phase.

In ultra-cold atomic gases, the strength of the interspecies and intra-species interaction can be varied by means of an external magnetic field (what is called Feshbach resonance method). This leads to the exploration of the whole phase diagram of the mixture [35, 29]. The Boson-Fermion mixture could be of two types. One in which the bosonic superfluid is the minority component, and the second in which the fermions are the minority component [36]. The miscibility and immiscibility is determined by interaction. It is also found that the Boson-Fermion phase diagram is known to admit, in addition to a fully mixed phase and a fully separated phase, also a third phase consisting of a pure fermions in equilibrium with a mixture of fermions and bosons[37]. Laser cooling can, lead to very low temperatures, of the order of 10^{-9}K (nano Kelvin). At these temperatures the Fermi gas will be degenerated, Bose gas will be a condensate, and the two systems can interact. There could be inter-gas and intra-gas interactions. However, the inter-gas interaction in a Fermi gas could be neglected due to Pauli Principle and the interactions between bosons, and bosons and fermions have to be taken into account. The boson-boson interaction is represented by g and the boson-fermion interaction is represented by h. The strength of both the interactions must be proportional to the S-wave scattering lengths.

To study the properties of a mixture of bosons and fermions at low temperature, Gross-Pitaevskii mean field equations for the boson distribution in the trap is solved by utilizing Thomas-Fermi Approximation to extract the density profile of boson and fermion components. How the condensates behave for different comparative values of h and g has been studied.

2. Theoretical Derivations

Assuming that the degenerate Fermi gas interacts with Bose condensate and the mixture is trapped in an external potential $V_{ext}(r)$, the atoms will interact by elastic collisions. At low temperatures, the atoms will have low kinetic energies and thus permit replacement of their short range interaction with a delta function. In the mean field description, the single particle wave function $\psi(r)$, assumed to describe all bosons in the gas, is governed by Gross Pitaevskii Equation.

$$\left\{-\frac{\hbar^2}{2M}\nabla^2 + V_{ext}(r) + gN_B|\psi(r)|^2\right\}|\psi(r)| = \mu\psi(r) \quad (3)$$

The Thomas Fermi Approximation exploits the fact that at low temperature, kinetic energy of the atom is so small that the kinetic energy operator $(-\frac{\hbar^2}{2M}\nabla^2 r)$ can be neglected. Hence Eq. (3) becomes,

$$(V_{ext}(r) + gN_B|\psi(r)|^2)\psi(r) = \mu\psi(r) \tag{4}$$

Dividing Eq. (4) by $\psi(r)$ yields,

$$V_{ext}(r) + gN_B|\psi(r)|^2 = \mu \tag{5}$$

Re-arranging Eq. (5) gives,

$$n_B(r) = N_B|\psi(r)|^2 = \frac{\mu - V_{ext}(r)}{g} \tag{6}$$

Where $N_B|\psi(r)|^2 = n_B(r)$ is bosonic density, $V_{ext}(r)$is the external confining potential and μ is the bosonic chemical potential (energy per particle). The value of μ is fixed by normalization condition. $\int d^3r.n_B(r) = N_B$, the total number of bosons. The harmonic oscillator potential is given by,

$$V_{ext}(r) = \frac{1}{2}\mathrm{M}\omega^2\mathrm{r}^2 \tag{7}$$

where r is the distance from the trap centre. μ is determined analytically to be

$$\mu = \left[\frac{15}{8\pi}Ng\left(\frac{M\omega^2}{2}\right)^{3/2}\right]^{2/5} \tag{8}$$

When N is very small, Thomas-Fermi Approximation gives a good approximation to the exact distribution of particles and to the single particle energy. For low kinetic energy, short range interaction potential is replaced by a delta function of strength g and h.

For the particles in the TFA (Thomas-Fermi Approximation), due to Pauli's exclusion principle, the atoms in the degenerate gas of fermions do not occupy a single state. Hence there is no equivalent of Gross Pitaevskii equation for fermions. Instead, the particles will be described by classical position and momenta. However, we use the quantum mechanical result that a volume in phase space d^3rd^3k can accommodate, $\frac{d^3rd^3k}{\hbar^3(2\pi)^3}$ fermions, i.e., if the local density $n_F(r)$ will have the wave numbers within the interval $0 \le k \le k_F(r)$. The fermions will experience a local potential,

$$V(r) = V_{ext}(r) + \mathrm{h}.n_F(r) \tag{9}$$

For the particles in motion for such a potential, it is possible to define a local Fermi vector $k_F(r)$ by,

$$\mathrm{E_F} = \frac{\hbar^2\mathrm{k_F(r)}^2}{2\mathrm{M}} + \mathrm{V(r)} \tag{10}$$

So that the volume of the local Fermi sea in k space is

$$\frac{4}{3}\pi k_F(r) = (2\pi)^3 n_F(r) \tag{11}$$

Local Fermi vector will be given by

$$k_F(r) = \left(6\pi^2 n_F(r)\right)^{1/3} \tag{12}$$

In low temperature limit, p-wave scattering can be neglected. The suppression of the s-wave scattering amplitude due to antisymmetry of the many body function implies that the spin polarized fermions may constitute a non-interacting gas; hence the energy density of fermionic component is given by the expression;

$$\frac{\hbar^2 k_F^2(r)}{2M} + V_{ext}(r) + h.n_B(r) = E_F \tag{13}$$

Re-writing Eq. (13) using Eq. (6) and Eq. (12) yields,

$$E_F = \frac{\hbar^2\left(6\pi^2 n_F(r)\right)^{2/3}}{2M} + \left[1 - \frac{h}{g}\right]V_{ext}(r) + \frac{h}{g}\mu \tag{14}$$

In isotropic traps, the trapping potential $V_{ext}(r)$ felt by bosonic component is equal to the local potential experienced by the fermionic component, i.e.

$$V_{ext}(r) = V(r)$$

or

$$\mu - g.n_B(r) = V_{ext}(r) + \mathrm{h}.n_F(r)$$

or

$$\mu = V_{ext}(r) + g.n_B(r) + \mathrm{h}.n_F(r) \tag{15}$$

In TFA, the density distribution for both components can be obtained by solving the coupled Eq. (14) and (15).

The mean occupation number of the single particle energy states with energy ε_n is given by

$$f(\varepsilon_n) = \frac{1}{\zeta^{-1}\varepsilon^{\beta\varepsilon_n} + a} \tag{16}$$

Where $\zeta = \varepsilon^{\beta\mu}$, is the fugacity, $\beta = \frac{1}{KT}$ and $a = \begin{cases} -1 & \text{Bose Einstein Statistics} \\ +1 & \text{Fermi Dirac Statistics} \\ 0 & \text{Maxwell} - \text{Boltzmann Statistics} \end{cases}$

In Fermi-Dirac statistics, the mean occupation number can become utmost one (Pauli's exclusion principle). Hence Eq. (16) becomes,

$$f(\varepsilon_n) = \frac{1}{\varepsilon^{\left(\frac{\varepsilon_n - \mu}{KT}\right)} + 1} \tag{17}$$

For harmonically trapped gases, density of states as a function of energy is given by[4],

$$g(\varepsilon) = \frac{\varepsilon^2}{2(\hbar\omega)^3} \tag{18}$$

The number of particles in the excited states can be calculated according to

$$N_F = \int_0^\infty f(\varepsilon) g(\varepsilon).\, d\varepsilon \tag{19}$$

Integrating Eq. (19) with $f(\varepsilon) = \begin{cases} 1 & \varepsilon > E_F \\ 0 & \varepsilon < E_F \end{cases}$

gives,

$$N_F = \int_0^{E_F} g(\varepsilon).\, d\varepsilon \tag{20}$$

Substituting Eq. (18) in Eq. (20) gives,

$$N_F = \int_0^{E_F} g \frac{\varepsilon^2}{2(\hbar\omega)^3}.\, d\varepsilon \tag{21}$$

Integrating Eq. (21) yields,

$$N_F = \frac{E_F^3}{6(\hbar\omega)^3} \tag{22}$$

The fermionic energy will be given by,

$$E_F = (6N_F)^{\frac{1}{3}}\hbar\omega \tag{23}$$

Combining Eq. (14) and (23) we get,

$$(6N_F)^{\frac{1}{3}}\hbar\omega = \frac{\hbar^2\left(6\pi^2 n_F(r)\right)^{2/3}}{2M} + \left[1 - \frac{h}{g}\right] V_{ext}(r) + \frac{h}{g}\mu \tag{24}$$

Combining Eq. (7), (8) and (24) yields,

$$(6N_F)^{\frac{1}{3}}\hbar\omega = \frac{\hbar^2\left(6\pi^2 n_F(r)\right)^{2/3}}{2M} + \left[1 - \frac{h}{g}\right]\frac{1}{2}\mathrm{M}\omega^2\mathrm{r}^2 + \frac{h}{g}\left[\frac{15}{8\pi} Ng \left(\frac{m\omega^2}{2}\right)^{3/2}\right]^{2/5} \tag{25}$$

Re-arranging Eq. (25) gives,

$$n_F(r) = \frac{\left\{\frac{2M}{\hbar^2}\left[\begin{array}{c}(6N_F)^{\frac{1}{3}}\hbar\omega - \left(1-\frac{h}{g}\right)\frac{1}{2}\mathrm{M}\omega^2 r^2 \\ -\frac{h}{g}\frac{15}{8\pi}N_B g\left(\frac{m\omega^2}{2}\right)^{3/2}\end{array}\right]^{\frac{2}{5}}\right\}^{\frac{3}{2}}}{6\pi^2} \tag{26}$$

Eq. (26) gives an expression for fermionic density.
Eq. (15) gives bosonic density $n_B(r)$, such that,

$$n_B(r) = \frac{\mu - V_{ext}(r) - h.n_F(r)}{g} \tag{27}$$

The strength of the boson-boson interaction g is chosen to give maximal overlap between the two atomic clouds. In order to have clouds of comparable sizes, we equate the Thomas Fermi expression for the radius of the Bose Condensate$(15N_B g/4\pi M\omega^2)^{\frac{1}{5}}$ and the radius of zero temperature Fermi gas$(48N_F)^{\frac{1}{6}}(\hbar/M\omega)^{\frac{1}{2}}$ such that,

$$(15N_B g/4\pi M\omega^2)^{\frac{1}{5}} = (48N_F)^{\frac{1}{6}}(\hbar/M\omega)^{\frac{1}{2}} \tag{28}$$

This gives;

$$g = \frac{21.1N_F^{\frac{5}{6}}}{N_B}.\hbar\omega a_0^3 \tag{29}$$

3. Parameters

Table 1 gives a list of parameters of the experiments with a ^{87}Rb–^{40}K boson-fermion mixture.

Parameters	Hamburg experiment[23]	Florence experiment [14,24,25]
mass of ^{87}Rb atom $m_B = 14.43\times 10^{-26}$ kg		
mass of ^{40}K atom $m_F = 6.636\times 10^{-26}$ kg		
s-wave scattering length (bosons ↔ bosons) $a_{BB} = (5.238\times 10^{-9} \pm 0.002)$ m		
s-wave scattering length (bosons ↔ fermions) $a_{BF} = -15.0\times 10^{-9}ma_{BF} = (-20.0\times 10^{-9} \pm 0.8)$ m		
radial trap frequency (bosons)	$\omega_{B,r} = 2\pi.257$ Hz	$\omega_{B,r} = 2\pi.215$ Hz
axial trap frequency (bosons)	$\omega_{B,z} = 2\pi.11.3$ Hz	$\omega_{B,z} = 2\pi.16.3$ Hz
radial trap frequency (fermions)	$\omega_{F,r} = 2\pi.\ 379$ Hz	$\omega_{F,r} = 2\pi.\ 317$ Hz
axial trap frequency (fermions)	$\omega_{F,z} = 2\pi.\ 16.7$ Hz	$\omega_{F,z} = 2\pi.\ 24.0$ Hz
number of bosons	$N_B = 10^6$	$N_B = 2\times 10^5$
number of fermions	$N_F = 7.5\times 10^5$	$N_F = 3\times 10^4$

Below is a list of parameters which have been used in the calculations. g, is chosen to give maximal overlap between the two atomic clouds. Other parameters used for calculations are;

$N_F = 10^3$
$N_B = 10^7$

$M_B = 1.45\times 10^{-25}Kg$
$M_F = 6.636\times 10^{-26}Kg$
$M = \frac{M_F M_B}{M_F + M_B} = 4.54559\times 10^{-26}Kg$
$\omega = 2\pi\times 216\mathrm{Hz}$
$g = 0.00066724\hbar\omega a_0^3$
$\hbar = \frac{h}{2\pi} = 1.0545\times 10^{-34}Js$
$a_0 = \left(\frac{\hbar}{M\omega}\right)^{\frac{1}{2}}$, h=g/2, h=g, and h=3g/2

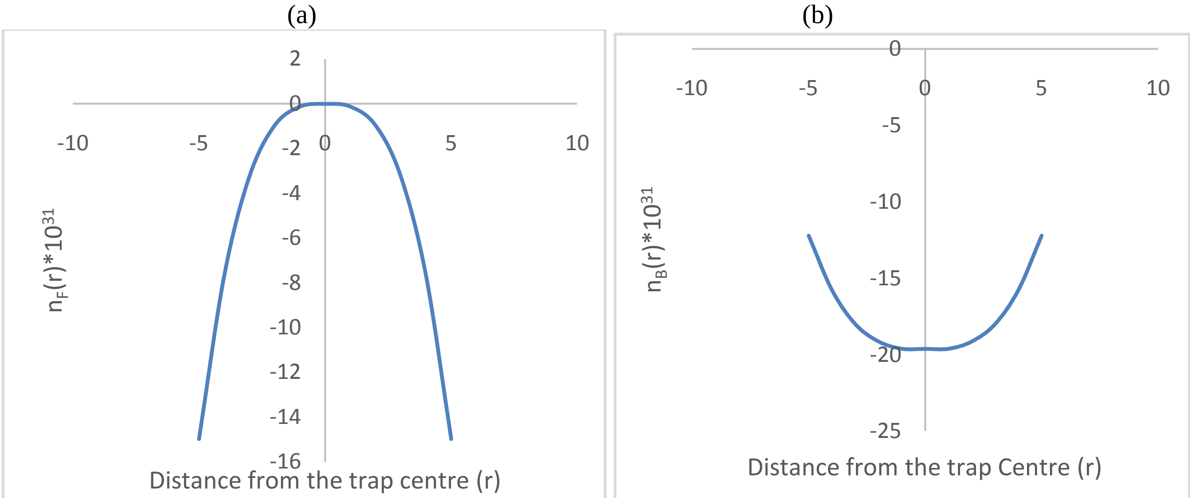
The density distribution of fermions is given by Eq. (26) and that of bosons is given by Eq. (27). Numerical values for $n_F(r)$ and $n_B(r)$can be obtained for different values of h and g from Eq. (26) and Eq. (27) respectively.

4. Results and Discussion

When h< g, the fermions experience a minimum potential at the center of the trap and therefore they populate around the center of the trap (r=0). Their number reduces as one moves away from the center. This is shown in the figure 1 (a). Bosons on the other end are expelled from the center of the trap. Their numbers increase as the distance from the trap center, is increased. This is shown in the figure 1(b). Since the number of fermions was small enough, they constituted a 'core' enclosed by the Bose condensate. The oscillation in the fermion density distribution near the trap center reflects the matter wave modulation in the outermost shell. In the center of the trap, the fermions in the lowest energy state experience a vanishing potential for h=0. The bosons are expelled from the trap center, minimizing their interaction energy by spreading in a shell around the fermionic bubble. The fermionic component is compressed, having a higher peak and density covering a smaller portion of the trapping volume. A similar behavior has been noted for bi-condensate systems [18, 19]. The two quantum gases are said to be truly interpenetrating. In this case a Bose condensate of about 10^7atoms may enclose roughly 10^3 Fermions. For $h < g$, the distribution is as shown in the Fig 1

Figure 1; (a) shows the density distribution for fermions and (b) bosons at zero kelvin temperature for $h < g \; (h = \frac{1}{2} g)$

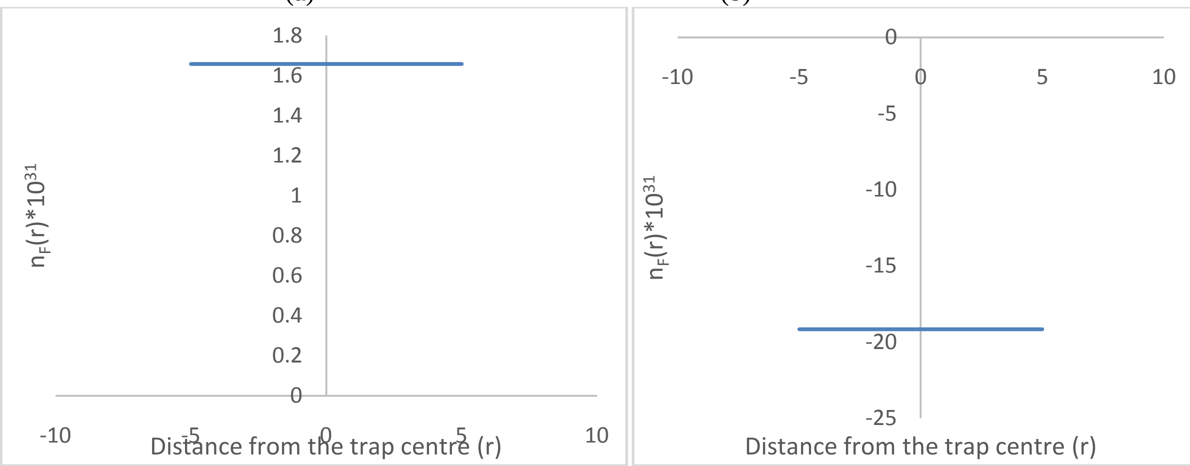
(a) (b)



Thomas Fermi theory provides of a functional form for kinetic energy of non-interacting electron gas .One of the essential features predicted in the Thomas-Fermi Approximation is the existence of a plateau of a constant fermionic density throughout the distribution. This phenomenon also appears in this quantum treatment. When h = g, the fermions have a constant density throughout the Bose condensate, as shown in the figure 2, falling towards zero outside.

Figure 2; (a) shows the density distribution for fermions and (b) bosons at zero kelvin temperature for h =g.

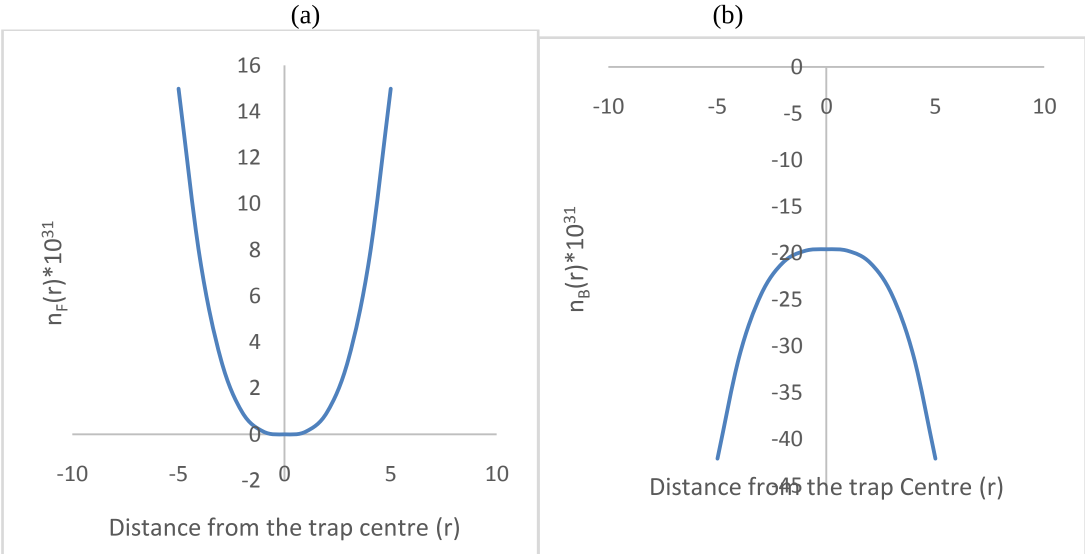
(a) (b)



When $h > g$, the effective potential for the fermions is that of an in inverted harmonic oscillator having a minimum at the edge of the Bose condensate, where the fermions localize as a "shell" wrapped round the condensate. If the outer part is composed of fermions, and the number of fermions exceeds the limit of the Bose Condensate, then the atoms in the inner core will experience a stronger confining potential. The distribution is as shown in the figure 3. The fermions have been expelled from the Centre of the trap Centre and their number increase as the distance from the trap is increased. We note that the semi-classical description gives a qualitatively correct description, in that it reliably predicts the phase separation.

Figure 3; (a) shows the density distribution for bosons and (b) fermions at zero kelvin temperature for h > g and h = 3g/2.

(a) (b)



We notice that as the bosons are expelled from the center of the trap, forming a 'mantle' around the fermions, the fermionic component is compressed, having a higher peak density and covering a smaller portion of the trapping volume. A similar behavior has been noted for bi-condensate systems [18, 19]. One of the essential features predicted in the Thomas-Fermi approximation is the existence of a 'plateau' of constant fermion density through the boson distribution for h = g as illustrated by Fig. 2. Such a phenomenon also appears in our quantum mechanical treatment, although with the parameters chosen it does not involve quite as many particles as obtained from the semi-classical calculations [2]. It is interesting to compare the above mentioned results with those obtained by treating the fermions in the Thomas-Fermi Approximation.

Depending on the relative magnitude of the types of interactions considered, one can obtain conditions for the kind of phase transitions and the phases (like super-fluid phase) that can exist in the mixture of bosons and fermions. Such studies will involve more complicated and advanced many-body theory. It will be important to consider the effect of attractive boson-fermion interactions, and that of the repulsive boson-fermion interactions on the density profiles of bosons, fermions and boson-fermion[38] mixture. Also some sort of interaction between fermions must be taken into account. A many-body theory in this regard is being developed and will be published later.

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