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Crystallization of Neutron Matter (Neutron Stars)

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Abstract: A neutron star is the collapsed core of a large (10 – 29 solar masses) stars. Neutron stars can be considered as reservoirs of high-density fermions, as these systems can be assumed to be the largest of its type in the universe. Neutron stars are the smallest and most dense stars of the size of 10Km radius, and their mass is roughly twice or more of the sun. We have calculated the energy per neutron in a neutronstar for low and high density neutron stars. It is found that the energy per neutron increases as the density of the neutron star increases for a given value of the scattering length.
Keywords: Crystallization, Fermions, Neutron matter, Neutron Rich Nuclei, Neutron Stars, Saturation Density, Scattering Length

1. Introduction

In the past, calculations [1] showed that the solidification pressure of the order of 5×10^{27} atmospheres can lead to the solidification of neutron matter in the vicinity of the density of neutron matter, $\rho_n = 5\times10^{14}\,gcm^{-3}$.As such, crystallization of neutron matter has been studied for high values of ρ_n as compared to density of high mass nuclei. Assuming different types of inter - particle interactions, and different spin states, such as the singlet states, $^1S_0$, $^1P_1$, $^1D_2$ and triplet states $^3s_1$, $^3p_0$, $^3p_1$, $^3p_2$, $^3D_1$, $^3D_2$, in which the two interacting particles can exist and also assuming solid ordered structures, such as body centred cubic (BCC), face centredcubic(FCC) and values of the neutron density in the range$\rho_n = 1.4X10^{15}\,gcm^{-3} to\ 5.237\ X10^{15}\,gcm^{-3}$. In general, the calculations showed that for densities $\rho_n = 1.5\ X\ 10^{15}\,gcm^{-3}$ or more, solid phase for cold matter can exist. The energies per particle (E/N) vary from structure to structure. For pure neutron matter for $\rho_n = 5.237\ X10^{15}\,gcm^{-3}$, the E/N for BCC is 1194.0 MeV, for HCP, E/N = 884.7Mev; and E/N =864.6 MeV for FCC. The saturation density of a nucleus $\approx 2.84 \times 10^{14}$ gcm^{-3} = ρ_s. Thus for the crystallization of neutron matter, the density should be roughly ten times, i.e, $\rho_n = \rho_s$.

It is by now known that neutron starscan be considered as reservoirs of high-densityfermions,and these systemsare perhaps the largest of its type in the universe. A neutron star is the collapsed core of a large (10-29 solar masses) star. Neutron stars are the smallest and most dense stars known to exist, typical size of a neutron star is 10km radius, but the mass could be twice or more than that of the sun [2]. Since the density of such neutron stars is almost ten times ($\rho_n \cong 10\rho_s$), the saturation density of a heavy nucleus, there exist strong nucleon-nucleon interactions between the neutrons, and consequently a number of different phases can occur.

Temperatures in the interior of the neutron stars fall below a billion degrees Kelvin in less than one year after the birth of the star [3]. Such temperatures may look high, but they are low compared with the characteristic energies such as the Fermi energy, which for nuclear density are of the order of 10-100 MeV, and this energy corresponds to temperatures of the order of 10^{11} x 10^{12} K (kT=energy). It is now understood that in the inner crust of neutron stars, the neutrons paired in a $^1S_0$state co-exist with a lattice of a neutron-rich nuclei; in fact superfluid neutrons co-exist with a crystal lattice of neutron-rich nuclei and an electron gas. To understand the properties of such systems, interaction between individual nucleons must be accurately known. As a first step a simple nucleon-nucleon interaction can be chosen to study the state of crystallization of neutron stars.

Another important property of matter is the density that determines the state of matter from gaseous to crystalline state. In the case of neutron star crust, it is well known that it is in a crystal state for a wide range [4]of mass densities and temperatures.

2. Dilute Neutron Gas (low density)

The theoretical study of dilute quantum gases goes back to 1950's and 1960's [5, 6, 7 and 8], but the experimental realization of such systems is a recent phenomenon. This has led to considerable insight into the properties of neutron matter.

At low energies, the effective interaction between two particles is determined by the S-wave scattering length, a. For two neutrons in the singlet spin state, the scattering length is -18.5fm, which is large in magnitude compared with the range of nuclear interactions $\approx\ 1.0fm$. For densities much less than $\frac{1}{a^3} \approx 10^{-4}\ fm^{-3}$(particle number density) which in mass density $\approx 10^{-4}\ fm^{-3} \times 1.67\times 10^{-24}$g $\approx\ \rho_n'\ 10^{-4} \times 10^{39} \times 1.67\times 10^{-24}$gcm^{-3} = 1.67 $\times\ 10^{11}$gcm^{-3}which is much less than the range of values ρ_n of the density of neutron matter in neutron stars. For the neutron matter whose density is of the order of ρ_n', the leading interaction contribution to the properties of the system can be calculated,in momentum space,in terms of an effective interaction of the form [2],

$$U_\circ = \frac{4\pi\hbar^2\ a}{m} \tag{1}$$

This corresponds to a delta function in co-ordinate space (hard-sphere interaction).

It is well known that the condition for a gas to be dilute is that the inter-particle spacing,r_s, must be large compared with the magnitude of the scattering length, a, of the particles, or since the Fermi wave number, k_F is proportional to, $1/\tau_s$, this condition is equivalent to k_F |a|$\ll 1$.

For neutron matter at low density, where the interaction is mainly S-wave, the BCS Theory can be used to calculate the energy of the system, the energy gap, and the thermodynamic properties such as the specific heat C_v, the entropy, S, and the transition temperature $T_c$to the superfluid state.

3. High Density Neutron Matter

At higher densities, there are larger uncertainties because additional terms in the neutron-neutron interaction (three neutron interactions etc.) become increasingly important and the increased density complicates the calculations. However, it is also possible that in neutron matter, three-neutron interactions may be suppressed because configurations in which three-neutrons are close together may be unlikely, since at least two of the neutrons must be in the same spin state. Thus when neutrons are very close under high pressure and high density, neutron crystallization is certainly possible, and this is what has been studied. Calculations have been done to get values of ($E/_N$) and how it varies with the density ρ_n , of the neutron star when it is in the crystalline state.

4. Theoretical Equations

The energy, E, of an assembly of crystalline neutron matter in the low density limit will be [9]

$$E/_N = \lambda_v\, \rho^{2/3} + \frac{(v-1)}{v}\frac{2\pi\hbar\rho a}{m} + \frac{4\pi\hbar^2 a}{\boldsymbol{m}} \tag{2}$$

where$\lambda_v = \frac{3\hbar^2}{10m}\left(\frac{6\pi^2}{v}\right)^{\frac{2}{3}}$, and$v = 2$, which is the intrinsic degrees of freedom for each fermion.

The energy, E, of an assembly of crystalline neutron matter in the high density limit ($\nu \to \infty, \lambda_v \to 0,$) will be [9]

$$\frac{E}{N} = \pi^2\, 2^{2/3}\frac{\hbar^2}{2m}\left[\frac{1}{\rho^{-1/3} - \rho_0^{-1/3}}\right]^2 + \frac{4\pi\hbar^2 a}{\boldsymbol{m}} \tag{3}$$

where$\rho_\circ = \frac{\sqrt{2}}{a^3}$ and a is the scattering length.

We have studied the variation of$\frac{E}{N}$ withρ both for low density and high density neutron stars, keeping '**a**' constant,ρ may be varied between 1.4x10^{15} g/cm^3 and 5.237x10^{15} g/cm^3 and 10.85x10^{15} g/cm^3, with a difference of 0.5; and **a** may be varied from -18.5fm to 1.0fm with a difference of 0.5. One value for ρ should be 3.7x10^{15}g/cm^3 since this is supposed to be the density of neutron matter at which there may be onset of the solid phase.We also calculated the value of$\frac{E}{N}$ for ρ =5x10^{14}g/cm^3, since at this density under a pressure of 5x10^{27}Atm, solidification of neutron matter occurs.

i) Low density Crystallization

Eq. (2) gives the energy E/N=$\in$, which can be written,

$$\in = \frac{p^2}{2m} \tag{4}$$

Now, if p is maximum fluctuation in the momentum of the particle in the state of crystallization, then the fluctuation Δx in the displacement will be,

$$\Delta x = \frac{\hbar}{\sqrt{2m \in}} \tag{5}$$

Substituting the values of $\hbar$, m and $\in$ in eq. (4), we obtain

$$\Delta x = 7.9 \times 10 =^{-9} \ cm. \tag{6}$$

Now, using the equation,

$$\Delta x \Delta p = \hbar \tag{7}$$

We get,

$$\Delta p = \frac{\hbar}{\Delta x} = 1.265 \times 10^{19} cm / \sec \tag{8}$$

ii) High density crystallization

Similarly from eq. (3) for high density condition, $\in = 3.113x10^{-14} erg$, from which we get,

$$\Delta x = 1.6x10^{-9}\text{cm} \tag{9}$$

And

$$\Delta p = 6.25x10^{-19}\text{gcms}^{-1} \tag{10}$$

=

5. Results and discussions

Table 1 below shows values of energy, fluctuations in position and momentum for low and high densities.

Table 1 The values of energy, position and momentum for low and high densities

	Low density	High density
ϵ	$1.435\text{x}10^{-15}$erg	$3.11\text{x}10^{-14}$erg
Δx	$7.9\text{x}10^{-9}$cm	$1.6\text{x}10^{-9}$cm
Δp	$1.265\text{x}10^{-19}$gcmsec^{-1}	$6.25\text{x}10^{-19}$gcmsec^{-1}

Table 2. Energy per particle and saturation density, ρ_s (hard-core radius a=2.1117Å)

ρ_s x 10^{15} g/cm^{-3}	**5.0**	**3.7**	**10.85**
$\boldsymbol{E/_N x10^{-30} J}$	**1.3**	**4.8**	**9.8**

Table 3 below shows energy per particle and saturation density for low and high densities at a constant radius, a=2.1117Å.

Table 3 Energy per particle and saturation density for low and high densities at a constant hard-core radius, a=2.1117Å

Density, ρ in terms of 10^{15}gcm^{-3}	Low density a = constant	High density a=constant
	$E/_N x10^{-30} J$	$E/_N x10^{-30} J$
1.4	2.5014	14.8
1.9	3.0019	18.1354
2.4	3.5874	21.1917
2.9	4.0699	24.0412
3.4	4.5254	26.7307
3.9	4.9589	29.2910
4.4	5.3739	31.7438
4.9	5.7749	34.1053
5.4	6.1614	36.3876

5.9	6.5359	38.6004
6.4	6.9004	40.7516
6.9	7.2559	42.8473
7.4	7.6024	44.8930
7.9	7.9409	46.8931
8.4	8.2729	48.8514
8.9	8.5982	50.7712
9.4	8.9174	52.6554
9.9	9.2410	54.5064
10.4	9.5301	56.3266
10.9	9.8332	58.1177
11.4	10.1317	59.8817

The figure below shows a graph of energy per particle against density.

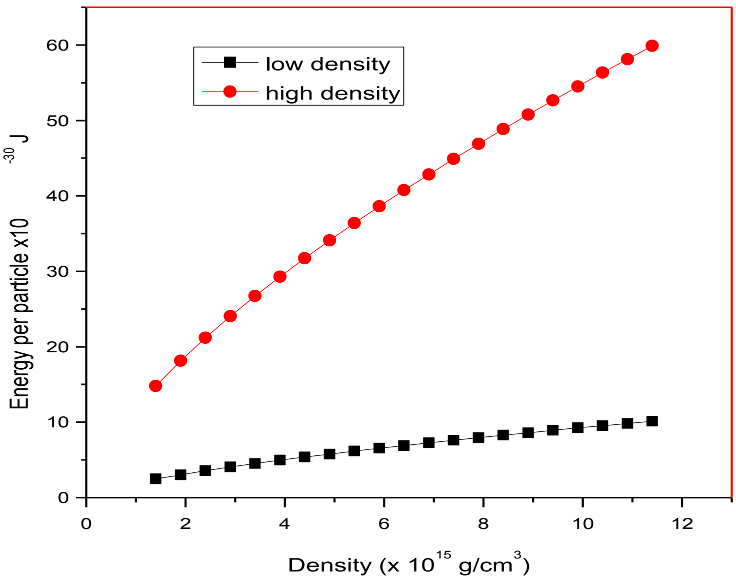

Fig. 1 Variation of energy per particle with density

5. Discussion of results

The table 2 shows that the energy E/N increases as the density increases, both for low density and high density neutron stars, keeping 'a' constant. This is exactly what it should be since increasing the density can lead to strong interactions resulting in the increase in energy of the system. This also confirms that under high pressures the system will have large density and huge amount of energy on crystallization.

From eqs. (4) to (10), it is clear that the fluctuation Δx in the position for low density system is larger than the corresponding value of Δx for the high density neutron star. It should be so since the high density system is more closely packed compared to the low density system. The fluctuation Δp in the momentum in high density system is large compared to the low density system, because high density system has more energy

Fig. 1 shows that for low density variation in E/N is almost linear with small gradient. For high density, the variation is large with large gradient. This is because in a high density system, the energies involved are large, Δx is large, and hence the variation has to be large.

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