International Journal of Latest Engineering Research and Applications (IJLERA) ISSN: 2455-7137

Volume – 03, Issue – 04, April 2018, PP – 20-24

E-cordial labeling of Path union of wheel related graphs

Mukund V. Bapat[1]
Hindale, Devgad, Sindhudurg, Maharashtra India

1. Abstract: In this paper we discuss e-cordial labeling of path union of wheel related graphs, Pm(G) where G is W_4, G_4, , W_5, G_5, flag of (W_4) given by Fl(W_4) ,Fl(W_5) ,Fl(G_4),Fl(G_5) and show that under certain condition they are e- cordial. We also show that the e-cordial labeling of path union is independent of vertex on G used to form the parh union.
Keywords: E-cordial, path union, fusion, edge, vertex, wheel. Gear graph.
Subject Classification: 05C78

2. Introduction

The graphs we discuss are simple, connected and finite. For terminology and definitions we depend on Graph Theory by Harary[4], Dynamic survey of graph labeling [3]. and West [6]

In 1997 Yilmaz and Cahit [5] introduced a weaker version of edge graceful labeling called E-cordial. The word cordial was used first time in this paper. Let G be a graph with vertex set V and edge set E. Let f be a function that maps E into {0,1} .Define f on V by f(v) = $\sum\{f(uv)/(uv)\in E\}$(mod 2).The function f is called as E cordial labeling if $|e_f(0)-e_f(1)|\leq 1$ and $|v_f(0)-v_f(1)|\leq 1$. Where $e_f(i)$ is the number of edges labeled with i = 0,1 and v_f (i) is the number of vertices labeled with i = 0,1. We also use $v_f(0,1)$ = (a,b) to denote the number of vertices labeled with 0 are a in number and that with 1 are b in number. Similarly $e_f(0,1)$=(x, y) to denote number of edges labeled with 0 are x in number and that labeled with 1 are y in number. A lot of work has been done in this type of labeling and the above mentioned paper gave rise to number of papers on cordial labeling. A graph that admits E-cordial labeling is called as E- cordial graph. Yilmaz and Cahit has shown that Trees T_n with n vertices and Complete graphs K_n on n vertices are E – cordial iff n is not congruent to 2 (modulo 4). Friendship graph $C_3^{(n)}$ for all n and fans F_n for n not congruent to 1 (mod 4).They observe that a graph with n vertices is not e-cordial if n≡ 2(mod 4).This observation is followed by graphs that we study next. One may refer A Dynamic survey of graph labeling for more details on completed work.

In this paper we discuss path unions of graphs obtained from W_4, W_5, G_4, G_5, and flag graphs. The e-cordial function f is independent of the vertex on G used to fuse on the vertex of path P_m to obtain path union.

3. Preliminaries:

1) **Fusion of vertex**. Let G be a (p,q) graph. Let u≠v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges.[6].If $u\in G_1$ and $v\in G_2$, where G_1 is (p_1,q_1) and G_2 is (p_2,q_2) graph. Take a new vertex w and all the edges incident to u and v are joined to w and vertices u and v are deleted. The new graph has p_1+p_2-1 vertices and $q_1 + q_2$ edges. Sometimes this is referred as u is identified with v.

2) **Path union of G** i.e.P_m(G) is obtained by taking a path P_m and m copies of graph G. Fuse a copy each of G at every vertex of path at given fixed point on G. It has mp vertices and mq +m-1 edges, where G is a (p,q) graph. If we change the vertex on G that is fused with vertex of Pm then we generally get a path union non isomorphic to earlier structure. In this paper we define a e-cordial function f that does not depends on which vertex of given graph G is fused to obtain path union. This allows us to obtain path union in which the same graph G is fused with vertices of P_m at different vertices of G, as our choice and the same function f is applicable to all such structures that are possible on P_m(G).

3) **Flag of a graph G**denoted by FL(G) is obtained by taking a graph G=G(p,q).At suitable vertex of G attach a pendent edge. It has p+1 vertices and q+1 edges. We show below in diagram flag of K_4.i.e.Fl (K_4)

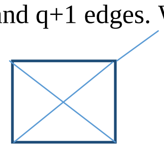

Fig 3.1 Fl(K_4)

4) **Wheel graph Wn.** Take acycleCn and a vertex w not on Cn. Join every vertex of Cn by an edge each to w.The resultant graph is wheel Wn. Sometimes the same graph is shown by W_{n+1}.It has n+1 vertices and 2n edges. The edges incident with w are called as pokes and vertexw as hub. The other edges that lie on cycle Cn are called as cycle edges.

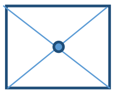

Fig 3.2 W_4

5) Gear graph G_n . It is obtained from wheel graph Wn .A new vertex is introduced on each cycle edge. Sometimes it is denoted by G_{n+1}.It has 2n+1 vertices and 3n edges.

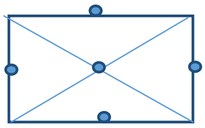

Fig 3.3 W_4

4. Main Results Proved:

Theorem 4.1: G= $P_m(W_4)$ is e-cordial iff m is not congruent to 2(mod 4).

Proof: Wetake a path $P_m=(v_1, v_2, \ldots v_m)$ and at each of it's vertex fuse a copy of W_4 at a fixed vertex of W_4.Define a function f:E(G)→ {0,1} as follows: Label of every edge on path P_m is '0'. Using f we get following three labeled copies of W_4 Type A , Type B and Type C. Of these type A only is E- cordial. We use them all suitably to obtain a labeled copy of G.

Type A

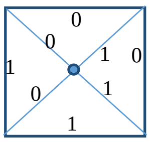

Fig 4.1 :$v_f(0,1)$ = (3,2), $e_f(0,1)$ = (4,4)

Type B

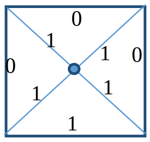

Fig 4.2 :$v_f(0,1)$ = (3,2), $e_f(0,1)$ = (3,5)

Type C

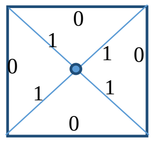

Fig 4.3 :$v_f(0,1)$ = (1,4), $e_f(0,1)$ = (4,4)

On P_m , for vertex v_1 fuse type A label. This is e-cordial follows from fig 4.1 On v_2 fuse type B label. At this stage label distribution is $v_f(0,1) = (6,4))$, $e_f(0,1) = (8,9)$. Note that it is not e-cordial. At v_3 fuse type C label. At this stage label distribution is $v_f(0,1) = (7,8))$, $e_f(0,1) = (13,13)$. At v_4 use Type B label. The label distribution is $v_f(0,1) = (10,10))$, $e_f(0,1) = (17,18)$. At v_5 use Type A label. Label distribution is $v_f(0,1) = (13,12))$, $e_f(0,1) = (22,22)$.

For i>5 Labeling is done in the following way.

At vertex v_i of path P_m fuse **type C** label if i≡ 2(mod 4).Label distribution is $v_f(0,1)$ = (14+10x,16+10x)), $e_f(0,1)$ = (27+18x,26+18x) for i = 4x+2.x = 2,3,..

At vertex v_i of path P_m fuse **type B** label if i≡ 3(mod 4). Label distribution is $v_f(0,1)$ = (17+10x,18+10x)), $e_f(0,1)$ = (31+18x,31+18x) for i= 4x+3; x= 2,3 ...

At vertex v_i of path P_m fuse **type A** label if i≡ 0(mod 4).Label distribution is $v_f(0,1)$ = (20+10x,20+10x)), $e_f(0,1)$ = (36+18x,35+18x) for i= 4x, x= 2,3,...

Atvertex v_i of path P_m fuse **type B** label if i≡ 1(mod 4).Label distribution is $v_f(0,1)$ = (23+10x,22+10x)), $e_f(0,1)$ = (40+18x,40+18x) for i= 4x+3.x = 2,3, ..

That is here onwards sequence of Type C, Type B, Type A, Type B is repeated . The function **f is not vertex sensitive** in the sense that which vertex on W_4 is used to form path union is not important. #

Theorem 4.2: G= $P_m(G_4)$ is e-cordial iff m is not congruent to 2(mod 4).

Proof: We take a path $P_m=(v_1, v_2, \ldots v_m)$ and at each of it's vertex fuse a copy of G_4 at a fixed vertex of G_4.Define a function f:E(G)→ {0,1} as follows: Label of every edge on path P_m is '0'. Using f we get following three labeled copies of G_4 Type A, Type B and Type C. Of these type A only is E- cordial. We use them all suitably to obtain a labeled copy of G

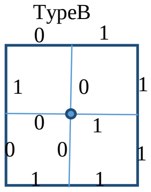

Fig 4.5 :$v_f(0,1)$ = (3,6), $e_f(0,1)$ = (6,6)

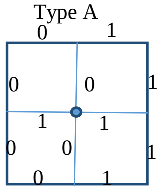

Fig 4.4 :$v_f(0,1)$ = (5,4), $e_f(0,1)$ = (6,6)

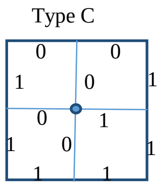

Fig 4.6 :$v_f(0,1)$ = (5,4), $e_f(0,1)$ = (5,7)

On P_m , for vertex v_1 fuse **type A** label, On v_2 fuse **type B**label. The label number distribution for $P_2(G_4)$ is$v_f(0,1)$ = (8,10)), $e_f(0,1)$ = (13,12). At this stage $P_m(G)$ is not e-cordial. At vertex v_3 fuse **type C** label. The label number distribution for $P_3(G_4)$ is$v_f(0,1)$ = (13,14)), $e_f(0,1)$ = (19,19).

At vertex v_4 use **type C** label. Label distribution is $v_f(0,1)$ = (18,18)), $e_f(0,1)$ = (25,26). For all i>4 and x = 1, 2, …We have; At vertex v_i on $P_m$fuse **type A** label if i ≡ 1(mod 4).Label distribution is $v_f(0,1)$ = (23+18x, 22+18x), $e_f(0,1)$ = (32+26x, 32+26x).; i = 4x + 1

At vertex v_i on P_m fuse **type B** label if i ≡ 2(mod 4).Label distribution is $v_f(0,1)$ = (26+18x, 28+18x), $e_f(0,1)$ = (39+26x, 38+26x).G is not e-cordial;i = 4x + 2

At vertex v_i on P_m fuse **type C** label if i ≡ 3(mod 4).Label distribution is $v_f(0,1)$ = (31+18x, 32+18x), $e_f(0,1)$ = (45+26x, 45+26x);i = 4x + 3

At vertex v_i on P_m fuse **type C** label if i ≡ 0(mod 4).Label distribution is $v_f(0,1)$ = (36+18x, 36+18x), $e_f(0,1)$ = (51+26x, 52+26x);.i = 4x + 4. Thus the graph is not e-cordial if m ≡ 2(mod 4).The function **f is not vertex sensitive** in the sense that which vertex on W_4 is used to form path union is not important. (one may use hub vertex or cycle vertex to obtain path union). The same f is applicable and gives us e-cordial labeling with same constraints.

Theorem 4.3: G= P_m (W_5) is e-cordial iff m is not congruent to 1(mod 2).

Proof: We take a path P_m=(v_1, v_2, …v_m) and at each of it's vertex fuse a copy of w_5 at a fixed vertex of W_5.Define a function f:E(G)→ {0,1} as follows: Label of every edge on path P_m is '0'.Using f we get following two labeled copies of W_5 Type A , Type B. We use them suitably to obtain a labeled copy of G

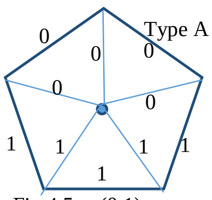

Fig 4.5 :$v_f(0,1)$ = (2,4), $e_f(0,1)$ = (5,5)

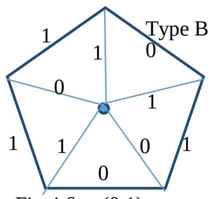

Fig 4.6 :$v_f(0,1)$ = (4,2), $e_f(0,1)$ = (4,6)

At vertex v_i fuse type A label if i= 1 (mod 2). At vertex v_i fuse type B label if i= 0 (mod 2) The label number distribution for $P_{2x+1}(G_4)$ is$v_f(0,1)$ = (2+6x,4+6x), $e_f(0,1)$ = (5+11x,5+11x). Wherei= 2x+1 , x= 0,1,2.. The label number distribution for $P_{2x}(G_4)$ is$v_f(0,1)$ = (6x,6x), $e_f(0,1)$ = (10+11x,11+11x). Wherei= 2x, x= 1,2..Thus the graph is e-cordial. The function f is not vertex sensitive in the sense that which vertex on W_5 is used to form path union is not important (we can use hub vertex or cycle vertex to obtain path union). The same f is applicable and gives us e-cordial labeling with same constraints.

Theorem 4.4: Let G'=fl(W_4).G= P_m(G') is e-cordial iff m is not congruent to 1(mod 2).

Proof: We take a path P_m=(v_1, v_2, …v_m) and at each of it's vertex fuse a copy of G' at a fixed vertex of G'. Define a function f: E(G)→ {0,1} as follows: Label of every edge on path P_m is '0'.Using f we get following two labeled copies of G' Type A , Type B. Of these none is E- cordial. We use them suitably to obtain a labeled copy of G.

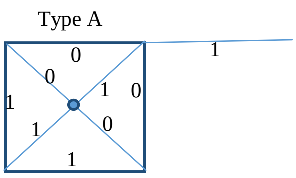

Fig 4.7 :$v_f(0,1)$ = (2,4), $e_f(0,1)$ = (4,5)

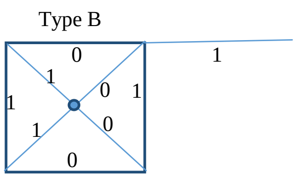

Fig 4.8 :$v_f(0,1)$ = (4,2), $e_f(0,1)$ = (4,5)

At vertex v_i of P_m use type A labeling if i≡1 (mod 2) and Type B labeling otherwise. The label number distribution is $v_f(0,1)$ = (2+6x,4+6x), $e_f(0,1)$ = (4+10x,4+10x) when m = 1+2x,x =0, 1, 2, .. if m is of type 2x, x=1,2,..$v_f(0,1)$ = (6x,6x), $e_f(0,1)$ = (9+10x,10+10x).Thus the graph is e-cordial iff m ≡ 0(mod2).

Theorem 4.5: Let G'=fl(G_4).G= P_m(G') is e-cordial iff m is not congruent to 1(mod 2).

Proof: We take a path P_m=(v_1, v_2, …v_m) and at each of it's vertex fuse a copy of G' at a fixed vertex of G'. Define a function f: E(G)→ {0,1} as follows:Label of every edge on path P_m is '0'. Using f we get following two labeled copies of G' Type A , Type B. Of these none is E- cordial. We use them suitably to obtain a labeled copy of G.

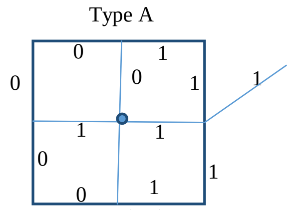

Fig 4.9 :$v_f(0,1)$ = (6,4), $e_f(0,1)$ = (5,7)

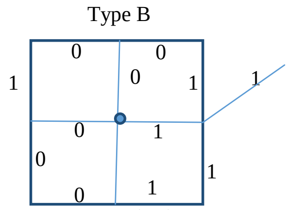

Fig 4.10: $v_f(0,1)$ = (4,6), $e_f(0,1)$ = (6,6)

At vertex vi of P_m use type A labeling if i≡1(mod 2) and Type B labeling otherwise. The label number distribution is $v_f(0,1)$ = (6+10x,4+10x), $e_f(0,1)$ = (5+13x,7+13x) when m = 1+2x,x =0, 1, 2, .. if m is of type 2x, x=1,2,..$v_f(0,1)$ = (10x,10x), $e_f(0,1)$ = (13x-1,13x).Thus the graph is e-cordial iff m ≡ 0(mod2).

Theorem 4.6: LetG= P_m(G_5) is e-cordial iff m is not congruent to 2(mod 4).

Proof: We take a path P_m= (v_1, v_2, …v_m) and at each of it's vertex fuse a copy of G_5 at a fixed vertex of G_5. Define a function f:E(G)→ {0,1} as follows: Label of every edge on path P_m is '0'.Using f we get following two labeled copies of G_5 Type A , Type B. Of these Type A is E- cordial. We use them suitably to obtain a labeled copy of G.

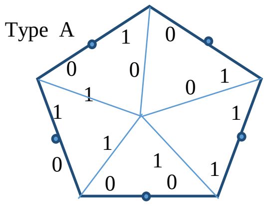

Fig 4.11: $v_f(0,1)$ = (5,6), $e_f(0,1)$ = (7,8)

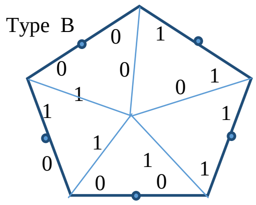

Fig 4.12 :$v_f(0,1)$ = (7,4), $e_f(0,1)$ = (7,8)

At vertex v_i of Pmfuse type A labeling if i ≡1, 3, 4(mod 4) and Type B labeling if i≡2(mod 4). The label number distribution is $v_f(0,1)$ = (5+22x,6+22x), $e_f(0,1)$ = ((7+32x,8+32x)) when m = 1+4x,x =0, 1, 2, .. If m is of type 4x, x=1,2,..Then,$v_f(0,1)$ = (22x,22x), $e_f(0,1)$ = (32x-1,32x).

If m is of type 4x+2, x=0,2,..$v_f(0,1)$ = (12+22x,10+22x), $e_f(0,1)$ = (15+32x,16+32x).

If m is of type 4x+3, x=0,2,..$v_f(0,1)$ = (17+22x,16+22x), $e_f(0,1)$ = (23+32x,24+32x).

Theorem 4.7: Let G= P_m(Fl(G_5)) is e-cordial iff m is not congruent to 2(mod 4).

Proof: We take a path P_m=(v_1, v_2, …v_m) and at each of it's vertex fuse a copy of Fl(G_5) at a fixed vertex of G_5. Define a function f:E(G)→ {0,1} as follows: Label of every edge on path P_m is '0'.Using f we get following labeled copy of Fl(G_5)Type A , Type B. Of these none is E- cordial. We use them suitably to obtain a labeled copy of G.

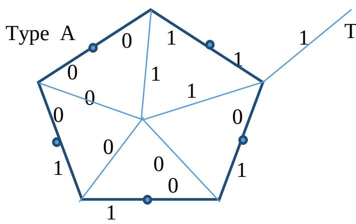

Fig 4.13: $v_f(0,1)$ = (6,6), $e_f(0,1)$ = (8,8)

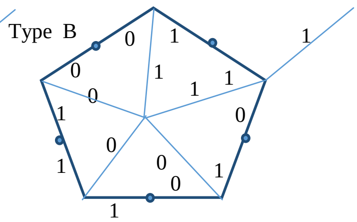

Fig 4.14 :$v_f(0,1)$ = (6,6), $e_f(0,1)$ = (7,9)

We Start with label A by fusing it with vertex v_i of path $P_m$for i≡ 1(mod 2) and Type B label for i≡ 0 (mod 2).The resultant graph is e-cordial.The label number distribution is $v_f(0,1)$ = (6m,6m) for all m and $e_f(0,1)$ =

((8+17x,8+17x)) when m = 1+2x,x =0, 1, 2, .. and $e_f(0,1)$ = ((17x-1,17x)) when m = 2x,x =1, 2, ..

Theorem 4.8: Let G= $P_m(Fl(W_5))$ is e-cordial iff m is not congruent to 2(mod 4).

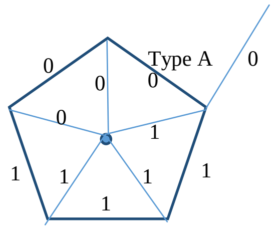

Fig 4.15 : $v_f(0,1)$ = (3,4), $e_f(0,1)$ = (5,6)

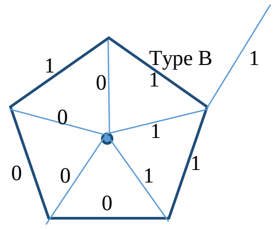

Fig 4.16 : $v_f(0,1)$ = (5,2), $e_f(0,1)$ = (5,6)

Proof: We take a path $P_m=(v_1, v_2, \ldots v_m)$ and at each of it's vertex fuse a copy of $Fl(W_5)$ at a fixed vertex of W_5. Define a function f: E(G)→ {0,1} as follows: Using f we get following labeled copy of $Fl(W_5)$: Type A , Type B. Of these Type A is E- cordial. We use them suitably to obtain a labeled copy of G.

Label of every edge on path P_m is '0'.At vertex v_i of P_m fuse type A labeling if i≡1,3,4(mod 4) and Type B labeling if i≡2(mod 4). The label number distribution is $v_f(0,1)$ = (3+14x,4+14x),$e_f(0,1)$ = ((5+24x,6+24x)) for m = 1+4x, x= 0,1,2,..The label number distribution is $v_f(0,1)$ = (8+14x,6+14x),$e_f(0,1)$ = ((11+24x,12+24x)) for m = 2+4x, x= 0,1,2,..The label number distribution is $v_f(0,1)$ = (11+14x,10+14x), $e_f(0,1)$ = ((17+24x,18+24x)),for m = 3+4x, x= 0,1,2,..The label number distribution is $v_f(0,1)$ = (14x,14x),for m = 4x, x= 1,2,..Further $e_f(0,1)$ = ((5+6(m-1),6+6(m-1)) for all m = 1, 2, ..

Conclusions:

In this paper we study path union of wheel related graphs and show that under certain conditions these graphs are e- cordial.

We show that

1) $P_m (W_4)$ is e-cordial iff m is not congruent to 2(mod 4).
2) $P_m(G_4)$ is e-cordial iff m is not congruent to 2(mod 4).
3) $P_m (W_5)$ is e-cordial iff m is not congruent to 1(mod 2).
4) G= $P_m (fl(W_4))$ is e-cordial iff m is not congruent to 1(mod 2).
5) G'=$fl(G_4)$.G= P_m(G') is e-cordial iff m is not congruent to 1(mod 2).
6) G= $P_m (G_5)$ is e-cordial iff m is not congruent to 2(mod 4).
7) G= $P_m (Fl(G_5))$ is e-cordial iff m is not congruent to 2(mod 4).
8) G= $P_m (Fl(W_5))$ is e-cordial iff m is not congruent to 2(mod 4).

It is important to note that all path unions $P_m(G)$ of graph G discussed above are not vertex sensitive in the sense that we can choose any vertex on G to form $P_m(G)$ and the resultant graph is e-cordial. We don't have to make any change in already defined function f.

References:

[1]. Bapat M.V. Ph.D. thesis, University Of Mumbai, 2004
[2]. E-cordial labeling of bull related graphs. 289-293, vol-3 issue 2,IJSAM.
[3]. Joe Gallian Dynamic survey of graph labeling 2016
[4]. Harary, Graph Theory, Narosapublishing, New Delhi
[5]. Yilmaz, Cahit, E-cordial graphs, Ars combina,46,251-256.
[6]. Introduction to Graph Theory by D. WEST, Pearson Education Asia.

