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Super Restrained Domination in the Corona of Graphs[1]

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Abstract: Let $G = (V(G), E(G))$ be a simple graph. A set $S \subseteq V(G)$ is a restrained dominating set if every vertex not in S is adjacent to a vertex in S and to a vertex in $V(G) \setminus S$. It is a super restrained dominating set if for every vertex $u \in V(G) \setminus S$, there exists $v \in S$ such that $N_G(v) \cap (V(G) \setminus S) = \{u\}$. The minimum cardinality of a super restrained dominating set in G, denoted by $\gamma_{spr}(G)$, is called the super restrained domination number of G. In this paper, we initiate the study of the concept and give the characterization of the corona of two graphs and give some important results.

Keywords: domination, restrained domination, super domination, super restrained domination
Mathematics Subject Classification: 05C69

I. INTRODUCTION

Graph Theory was born in 1736 with Euler's paper in which he solved the Konigsberg bridge problem [1]. Domination in graph was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [2]. However, it was not until following an article by Ernie Cockayne and Stephen Hedetniemi [3], that domination became an area of study by many. As Hedetniemi and Laskar [4] note, the domination problem was studied from the 1950s onwards, but the rate of research on domination significantly increased in the mid-1970s. The dominating set problem concerns testing whether $\gamma(G) \leq k$ for a given graph G and input k; it is a classical NP-complete decision problem in computational complexity theory [5]. One type of domination parameter is the restrained domination number in a graph. This was introduced by Telle and Proskurowski [6] indirectly as a vertex partitioning problem. Restrained domination in graphs can be read in the paper of Domke et.al. [7]. Other variant of restrained domination in graphs can be read in [8,9,10,11,12,13]. The super dominating sets in graphs was initiated by Lemanska et.al. [14]. Variation of super domination in graphs can be read in the paper of Baldado et.al [15]. Motivated by these parameters, we initiate the study of super restrained domination in graphs.

Let $G = (V(G), E(G))$ be a connected simple graph and $v \in V(G)$. The neighborhoodof v is the set $N_G(v) = N(v) = \{ u \in V(G): uv \in E(G) \}$. If $S \subseteq V(G)$, then the open neighborhood of S is the set $N_G(S) = N(S) = \cup_{v \in S} N_G(v)$. The closed neighborhood of S is $N_G[S] = N[S] = S \cup N(S)$. A subset S of $V(G)$ is a dominating set of G if for every $v \in (V(G) \setminus S)$, there exists $x \in S$ such that $xv \in E(G)$, i.e., $N[S] = V(G)$. The domination number $\gamma(G)$ of G is the smallest cardinality of a dominating set of G.

A set $S \subseteq V(G)$ is a restrained dominating set if every vertex not in S is adjacent to a vertex in S and to a vertex in $V(G) \setminus S$. Alternately, a subset S of $V(G)$ is a restrained dominating set if $N[S] = V(G)$ and $\langle V(G) \setminus S \rangle$ is a subgraph without isolated vertices. The restrained domination number of G, denoted by $\gamma_r(G)$, is the minimum cardinality of a restrained dominating set of G. A set $D \subset V(G)$ is called a super dominating set if for every vertex $u \in V(G) \setminus D$, there exists $v \in D$ such that $N_G(v) \cap (V(G) \setminus D) = \{u\}$. The super domination number of G is the minimum cardinality among all super dominating set in G denoted by $\gamma_{sp}(G)$. A restrained dominating set S is a super restrained dominating set in a graph G if for every vertex $u \in V(G) \setminus S$, there exists $v \in S$ such that $N_G(v) \cap (V(G) \setminus S) = \{u\}$. The minimum cardinality of a super restrained dominating set in G, denoted by $\gamma_{spr}(G)$, is called the super restrained domination number of G. In this paper, we initiate the study of the concept and give some important results. For general concepts we refer the reader to [16].

II. RESULTS

We are needing the following remarks for our subsequent results.

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Remark 2.1 Let G be a connected graph of order $n \geq 3$. Then

(i) $\gamma_{spr} \in \{2,3,..,n-3,n-2,n\}$.
(ii) $\gamma(G) \leq \gamma_r(G) \leq \gamma_{spr}(G)$.
Remark 2.2 The super restrained dominating set is a super dominating set and a restrained dominating set.

It is worth mentioning that the upper bound in Remark 2.1 is sharp. For example, $\gamma_{spr}(K_n) = n$. The next result says that the value of the parameter $\gamma_{spr}(G)$ ranges over all positive integers, $2,3,\ldots,n-3,n-2,n$.

Theorem 2.3 Given positive integers k and n such that $n \geq 4$ and $k \in \{2,3,\ldots,n-2,n\}$, there exists a connected graph G with $|V(G)| = n$ and $\gamma_{spr}(G) = k$.

Proof. Consider the following cases:

Case1. Suppose $k = 2$.

Let $G = C_n$ and $n = 4$. Clearly, $\gamma_{spr}(G) = 2 = k$ and $|V(G)| = 4 = n$.

Case2. Suppose $3 \leq k < n-2$.

Let $G \cong H \circ P_1$ where H is a nontrivial connected graph. Let $n = 2k$ and $V(H) = \{a_1, a_2, \ldots, a_k\}$. Then the set $S = \cup_{v\in V(H)} V(P_1^v) = \{V(P_1^{a_1}), V(P_1^{a_2}), \ldots, V(P_1^{a_k})\}$ is a γ_{spr}*-set* in G. Thus, $\gamma_{spr}(G) = |S| = k$ and

$$\begin{aligned} |V(G)| = |V(H \circ P_1)| &= |V(H) \cup (\cup_{v\in V(H)} V(P_1^v))| \\ &= |V(H)| + |\cup_{v\in V(H)} V(P_1^v)| \\ &= |V(H)| + |S| \\ &= k + k = 2k = n. \end{aligned}$$

Case3. Suppose $k = n-2$.

Let $G \cong K_2 \circ H$, $V(K_2) = \{a, b\}$ and H be a graph of order $m \geq 1$. Let $n = 2(m+1)$. Then set $S = V(H^a) \cup V(H^b)$ is a γ_{spr}*-set* in G. Thus, $\gamma_{spr}(G) = |S| = |V(H^a)| + |V(H^b)| = m + m = 2(m+1) - 2 = n - 2 = k$ and $|V(G)| = |V(K_2 \circ H)| = |V(K_2) \cup (\cup_{v\in V(K_2)} V(H^v))| = |VK_2| + |V(K_2)||V(H)| = 2 + 2m = n$.

Case4. Suppose $k = n$.

Let $G = K_n$. Then the set $V(K_n)$ is a γ_{spr}*-set* in G. Thus, $\gamma_{spr}(G) = n = k$ and $|V(G)| = n$.

This proves the assertion. ■

Theorem 2.4 Given positive integers k, m and $n \geq 6$ such that $2 \leq k \leq m \leq n-2$, there exists a connected graph G with $|V(G)| = n, \gamma_{spr}(G) = m$, and $\gamma_r(G) = k$.

Proof. Let $G \cong H \circ I$ and consider the following cases:

Case1. Suppose $m = n-2$.

Let $k = 2, H = P_2$, and $I = K_p$ where $m = 2p$. The set $S = V(G) \setminus V(H)$ is a γ_{spr}*-set* and $R = V(H)$ is a γ_r*-set* in G. Thus, $\gamma_{spr}(G) = |S| = |V(G)| - |V(H)| = |V(P_2 \circ K_p)| - V(P_2) = (2+2p) - 2 = m$ and $\gamma_r(G) = |R| = |V(H)| = |V(P_2)| = 2 = k$. Further, $|V(G)| = 2 + 2p = 2 + m = n$.

Case2. Suppose $m < n-2$.

If $k = m$, then let H be a nontrivial connected graph and $I = P_1$. Let $n = 2m$ and $V(H) =\{a_1, a_2, \ldots, a_k\}$. Then the set $S = \cup_{v\in V(H)} V(I^v) = \cup_{v\in V(H)} V(P_1^v) =\{V(P_1^{a_1}), V(P_1^{a_2}), \ldots, V(P_1^{a_k})\}$ is a γ_{spr}*-set* in G and a γ_r*-set* in

G. Thus, $\gamma_{spr}(G) = |S| = k = m$ and $\gamma_r(G) = |S| = k$. Further, $|V(G)| = |V(H \circ P_1)| = |V(H)| + |\bigcup_{v \in V(H)} V(P_1^v)| = k + k = 2k = 2m = n$.

If $k < m$, then let H be nontrivial connected graph of order k and $I = P_3$ with $3k = m$ and $n = k + m$. Then the set $S = \bigcup_{v \in V(H)} V(I^v)$ is a γ_{spr}-set in G and $R = V(H)$ is a γ_r-set in G. Thus, $\gamma_{spr}(G) = |S| = \left|\bigcup_{v \in V(H)} V(I^v)\right| = |V(H)||V(I)| = k \cdot 3 = m$ and $\gamma_r(G) = |R| = |V(H)| = k$. Further, $|V(G)| = |V(H \circ I)| = |V(H)| + |\bigcup_{v \in V(H)} V(I^v)| = k + m = n$. This proves the assertion. ■

The following results, follows from Theorem 2.4.

Corollary 2.5 The difference $\gamma_{spr} - \gamma_r$ can be made arbitrarily large.

Proof. Let n be a positive integer. Then there exists a connected graph G such that $\gamma_{spr}(G) = n + k$ and $\gamma_r(G) = k$ By Theorem 2.4. Thus, $\gamma_{spr}(G) - \gamma(G) = n$, showing that $\gamma_{spr} - \gamma$ can be made arbitrarily large. ■

Remark 2.6 The $\gamma_{sp}(K\_n) = n - 1$ for all $n \geq 2$.

Let G and H be graphs of order m and n, respectively. The corona of two graphs G and H is the graph $G \circ H$ obtained by taking one copy of G and m copies of H, and then joining the ith vertex of G to every vertex of the ith copy of H. The join of vertex v of G and a copy H^v of H in the corona of G and H is denoted by $v + H^v$.

We need the following results for the characterization of the super restrained dominating set in the corona of two graphs.

Lemma 2.7 Let $G = K_1 + H$ where H is a nontrivial connected graph and $x \in V(G)$. Then $V(G) \setminus \{x\}$ is a super dominating set of G.

Proof. Suppose that $G = K_1 + H$ where H is a nontrivial connected graph and $x \in V(G)$. Let $S = V(G) \setminus \{x\}$. Then $x \in V(G) \setminus S$ and $N(v) \cap (V(G) \setminus S) = \{x\}$ for some $v \in S$. This shows that S is a super dominating set.■

Lemma 2.8 Let G and H be nontrivial connected graphs. Then a nonempty subset S of $V(G \circ H)$ is a super restrained dominating set in $G \circ H$ if $S = (\bigcup_{v \in V(G)} V(H^v))$.

Proof. Suppose that $S = (\bigcup_{v \in V(G)} V(H^v))$. Then S is a dominating set of $V(G \circ H)$. Let $y \in V(G)$. Then $y \in V(G \circ H) \setminus S$. Since G is nontrivial connected graph, there exists $z \in V(G \circ H) \setminus S, (z \neq y)$ such that $yz \in E(G \circ H)$. Since S is dominating, there exists $x \in S$, such that $xy \in E(G \circ H)$. Thus, S is a restrained dominating set in $G \circ H$ by definition. Let $u \in V(G \circ H) \setminus S$. Since S is dominating, there exists $v \in S$ such that $u \in N_{G \circ H}(v)$, that is, $u \in N_{G \circ H}(v) \cap V(G)$ and $v \in V(H^u)$. Now, let $u' \neq u$ such that $u' \in N_{G \circ H}(v) \cap V(G)$. Then $u' \in N_{G \circ H}(v)$ and $u' \in V(G)$. This implies that $v \in V(H^{u'})$. Since $u \neq u'$ and $u, u' \in V(G)$, it follows that $V(H^u) \neq V(H^{u'})$. Thus, $v \in V(H^u) \neq V(H^{u'})$, that is, $u' \notin N_{G \circ H}(v)$. Hence, $u' \notin N_{G \circ H}(v) \cap V(G)$ contrary to our assumptions. Therefore, $u' = u$, that is, $N_{G \circ H}(v) \cap V(G) = \{u\}$. Accordingly, S is a super restrained dominating set in $G \circ H$. ■

Lemma 2.9 Let G and H be nontrivial connected and noncomplete connected graphs respectively. Then a nonempty subset S of $V(G \circ H)$ is a super restrained dominating set in $G \circ H$ if $S = V(G) \cup (\bigcup_{v \in V(G)} S_v)$ for each $v \in V(G)$ where S_v is a super restrained dominating set of H^v.

Proof. Let $u \in V(H^v) \setminus S_v$ for each $v \in V(G)$. Since S_v is a restrained dominating set of H^v, $(\bigcup_{v \in V(G)} S_v)$ is a restrained dominating set of $G \circ H$ and hence $S = V(G) \cup (\bigcup_{v \in V(G)} S_v)$ is a restrained dominating set of $G \circ H$.

Furthermore, since S_v is a super dominating set of H^v for each $v \in V(G)$, there exists $x \in S_v$ such that $N_{H^v}(x) \cap (V(H^v) \setminus S_v) = \{u\}$ for each $u \in V(H^v) \setminus S_v$. Thus, $u \in V(H^v) \setminus S_v \subset V(G \circ H) \setminus S$ for each $v \in V(G)$, and for each u, there exists $x \in S_v \subset S$ such that $N_{G \circ H}(x) \cap (V(G \circ H) \setminus S) = \{u\}$. This implies that S is a super dominating set of $G \circ H$. Accordingly, S is a super restrained dominating set of $G \circ H$. ■

Lemma 2.10 Let G and H be nontrivial connected graphs. If $V(H^u)$ is a super dominating set of $u+H^u$ for all $u \in V(G)$ then $\bigcup_{u\in V(G)} V(H^u)$ is a super restrained dominating set of $G \circ H$.

Proof. Suppose that $V(H^u)$ is a super dominating set of $u+H^u$ for all $u \in V(G)$. Then $\bigcup_{u\in V(G)} V(H^u)$ is a dominating set of $G \circ H$. This implies that for each $u \in V(G) = V(G\circ H) \setminus (\bigcup_{u\in V(G)} V(H^u))$ there exists $z \in (\bigcup_{u\in V(G)} V(H^u))$ such that $u \in N_{G\circ H}(z) \cap V(G)$. Since G is nontrivial connected graph, let $u^{'} \in V(G)$ such that $u' \neq u$. Suppose that $u^{'} \in N_{G\circ H}(z) \cap V(G)$. Then $u^{'} \in N_{G\circ H}(z)$. This implies that $z \in V(H^{u^{'}})$. Since $u \neq u$ and $u, u^{'} \in V(G)$, it follows that $V(H^u) \neq V(H^{u^{'}})$. Thus, $z \in V(H^u) \neq V(H^{u^{'}})$, that is, $u' \notin N_{G\circ H}(z)$. Hence, $u^{'} \notin N_{G\circ H}(z) \cap V(G)$ contrary to our assumptions. Therefore, $u' = u$, that is, $N_{G\circ H}(v) \cap V(G) = \{u\}$. Thus, $(\bigcup_{u\in V(G)} V(H^u))$ is a super dominating set in $G \circ H$. Now, let $x \in V(G)$. Since G is nontrivial connected graph, there exists $x^{'} \in V(G) \setminus \{x\}$ such that $xx^{'} \in E(G\circ H)$ and $xv \in E(G\circ H)$ for some $v \in (\bigcup_{u\in V(G)} V(H^u))$. Thus, $(\bigcup_{u\in V(G)} V(H^u))$ is a restrained dominating set of $G \circ H$. By Remark 2.2 $(\bigcup_{u\in V(G)} V(H^u))$ is a super restrained dominating set of $G \circ H$. ■

Lemma 2.11 Let $G\ and\ H$ be nontrivial connected graphs. Then a proper subset $S\ of\ V(G\circ H)$ is a super restrained dominating set in $G\circ H$ if $S = S_G \cup (\bigcup_{v\in V(G)} V(H^v))$ where $S_G \subset V(G), |V(G) \setminus S_G| \geq 2$ and the subgraph induced by $V(G) \setminus S_G$ has no isolated vertices.

Proof. Suppose that $S = S_G \cup (\bigcup_{v\in V(G)} V(H^v))$ where $S_G \subset V(G), |V(G) \setminus S_G| \geq 2$ and the subgraph induced by $V(G) \setminus S_G$ has no isolated vertices. Then S is a dominating set of $G \circ H$. Let $u \in V(G\circ H) \setminus S$. Then $V(G\circ H) \setminus S = V(G\circ H) \setminus (S_G \cup (\bigcup_{v\in V(G)} V(H^v))) = V(G) \setminus S_G$. This implies that $u \in V(G) \setminus S_G$. Then there exists $v \in V(H^u) \subset S$ such that $N_{G\circ H}(v) \cap (V(G\circ H) \setminus S) = (N_{H^u}(v) \cup \{u\}) \cap (V(G) \setminus S_G) = \{u\}$. Thus, \$$S$\$ is a super dominating set of $G \circ H$. Since $|V(G\circ H) \setminus S| = |V(G) \setminus S_G| \geq 2$ and the subgraph induced by $V(G\circ H) \setminus S$ has no isolated vertices, it follows that S is a restrained dominating set of $G \circ H$. Accordingly, S is a super restrained dominating set of $G \circ H$. ■

The next result is the characterization of the super restrained dominating set in the corona of two graphs.

Theorem 2.12 Let G and H be nontrivial connected graphs. Then a proper subset S of $V(G\circ H)$ is a super restrained dominating set in $G \circ H$ if and only if one of the following statements holds:

(i) $S = \bigcup_{v\in V(G)} V(H^v)$.
(ii) $S = S_G \cup (\bigcup_{v\in V(G)} V(H^v))$ where $S_G \subset V(G), |V(G) \setminus S_G| \geq 2$ and the subgraph induced by $V(G) \setminus S_G$ has no isolated vertices
(iii) $S = V(G) \cup (\bigcup_{v\in V(G)} S_v)$ where S_v is a super restrained dominating set of H^v.

Proof. Suppose that S of $V(G\circ H)$ is a super restrained dominating set of $G \circ H$. Then S is a super dominating set and a restrained dominating set by Remark 2.2. Let $u \in V(G\circ H) \setminus S$ and consider the following cases:

Case1. If $u \in V(G)$, then $V(G\circ H) \setminus S \subseteq V(G)$. If $V(G\circ H) \setminus S = V(G)$, then $S = \bigcup_{v\in V(G)} V(H^v)$. This proves statement (i). If $V(G\circ H) \setminus S \subset V(G)$ then there exists $x \in V(G)$ such that $x \neq V(G\circ H) \setminus S$ and so $x \in S$. Let $x \in S_G$ where $S_G = V(G) \cap S$. Then $S = S_G \cup (\bigcup_{v\in V(G)} V(H^v)$. Now,

$$V(G\circ H) \setminus S = V(G\circ H) \setminus (S_G \cup \left(\bigcup_{v\in V(G)} V(H^v)\right) = V(G) \setminus S_G.$$

Since S is a restrained dominating set of $G \circ H$, it follows that $|V(G) \setminus S_G| \geq 2$ and the subgraph induced by $V(G) \setminus S_G$ has no isolated vertices. This proves statement (ii).

Case2. If $u \notin V(G)$, then $u \in V(H^v)$, where $v \in V(G)$ and hence $u \in (\bigcup_{v\in V(G)} V(H^v))$. This implies that $V(G\circ H) \setminus S \subseteq \left(\bigcup_{v\in V(G)} V(H^v)\right)$. *If* $V(G\circ H) \setminus S = (\bigcup_{v\in V(G)} V(H^v))$, then $S = V(G)$. Thus, for each $u \in V(G\circ H) \setminus S$, there exists $v \in S$ such that $N_{G\circ H}(v) \cap (V(G) \setminus S) = V(H^v)$. Since H is nontrivial, $N_{G\circ H}(v) \cap (V(G) \setminus S) \neq \{u\}$ contrary to our assumption that S is a super dominating set of $G \circ H$. Thus, $V(G\circ H) \setminus S \neq (\bigcup_{v\in V(G)} V(H^v))$ and so $V(G\circ H) \setminus S \subset (\bigcup_{v\in V(G)} V(H^v))$. This implies that there exists

$x \in (\bigcup_{v\in V(G)} V(H^v))$ such that $x \notin V(G \circ H) \setminus S$ and hence $x \in S$. Let $x \in S_v \subset S$ where $v \in V(G)$. Then $S = V(G) \cup (\bigcup_{v\in V(G)} S_v)$. Suppose that S_v is not a super dominating set of H^v for each $v \in V(G)$. Then there exists $u \in V(H^v) \setminus S_v$ such that $N_{H^v}(x) \cap (V(H^v) \setminus S_v) \neq \{u\}$ for all $x \in N_{H^v}(u) \cap S_v$. Thus, there exists $u \in V(G \circ H) \setminus S$ such that $N_{G\circ H}(x) \cap (V(G \circ H) \setminus S) \neq \{u\}$ for all $x \in N_H(u) \cap S$ contrary to our assumption that S is a super dominating set of $G \circ H$. Thus, S_v must be a super dominating set of H^v. Suppose that S_v is not a restrained dominating set of H^v. Then there exists $u \in V(H^v) \setminus S_v$ such that $ux \notin E(H^v)$ for all $x \in S_v$ or $uy \notin E(H^v)$ for all $y \in V(H^v) \setminus \{u\}$. Thus, there exists $u \in V(G \circ H) \setminus S$ such that $ux \in E(G \circ H)$ for all $x \in S$ or $uy \notin E(G \circ H)$ for all $y \in V(G \circ H) \setminus \{u\}$. In either case, S is not a restrained dominating set of $G \circ H$ contrary to our assumption. Thus, S_v must be a restrained dominating set of H^v. This implies that S_v is a super restrained dominating set of H^v. This proves statement (iii).

For the converse, suppose that statement (i) is satisfied. In view of Lemma 2.8, S is a super restrained dominating set of $G \circ H$. Next, suppose that statement (ii) is satisfied. Then S is a super restrained dominating set of $G \circ H$ by Lemma 2.11. Finally, suppose that statement (iii) is satisfied. Then by Lemma 2.9, S is a super restrained dominating set of $G \circ H$. ■

The following result is an immediate consequence of Theorem 2.12.

Corollary 2.13 Let G and H be a nontrivial connected graphs where H is a complete graph. Then $\gamma_{spr}(G \circ H) = |V(G)||V(H)|$.

Proof. Let $S = \bigcup_{v\in V(G)} V(H^v)$. The S is a super restrained dominating set in $G \circ H$ by Theorem 2.12 (i). Thus, $\gamma_{spr}(G \circ H) \leq |S| = |\bigcup_{v\in V(G)} V(H^v)| = |V(G)||V(H)|$.

Let S be a γ_{spr}-set in $G \circ H$. Then $|S| = \left|\bigcup_{u\in V(G)} S_u\right|$ for some $S_u \subseteq V(H^u)$ for each $u \in V(G)$. If $S_u \neq V(H^u)$, then let $u' \in V(H^u) \setminus S_u$ for each $u \in V(G)$. Since $u' \notin S, u' \in V(G \circ H) \setminus S$. Since H is complete, for each $v \in S_u, uv, u'v \in E(u + H^u)$. Thus, for each $u' \in V(G \circ H) \setminus S$ there exists $v \in S_u \subset S$ such that $N_{G\circ H}(v) \cap (V(G \circ H) \setminus S) = (N_{H^u}(v) \cup \{u\}) \cap (V(H^u) \setminus (S_u \cup \{u\})) = \{u, u'\}$. This implies that S is not a super dominating set of $G \circ H$. Thus, $S_u = V(H^u)$ and

$$\gamma_{spr}(G \circ H) = |S| = |\bigcup_{u\in V(G)} S_u| = |\bigcup_{u\in V(G)} V(H^u)| = |V(G)||V(H)|. \quad ■$$

III. CONCLUSION

A super restrained dominating set is a new variant of domination in graphs and hence this paper is a contribution to the development of domination theory in general. Since this is new, further investigations on binary operations and bounds of this parameter must be done to come up with more substantial results. Thus, the join, lexicographic, Cartesian products of two graphs of super restrained dominating sets are recommended for further study. Moreover, domination in graphs is rich with immediate applications in the real world such as routing problems in internets, problems in electrical networks, data structures, neural and communication networks, protection and location strategies and others. The super restrained domination in graphs is not far from these applications.

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