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Product cordial labeling of path union graphs related to C_3

Mukund V. Bapat[1]

Abstract: In this paper we study path union obtained by fusing a fixed vertex of graph G with each vertex of a path P_m.We take G = Flag C_3 , C_3^+, bull C_3, C_3^{++} etc.

Keywords: labeling, cordial, product, bull graph, crown, tail graph.

Subject Classification: 05C78

2. Introduction:

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [9], A dynamic survey of graph labeling by J.Gallian [6] and Clark, Holton [7]. I.Cahit introduced the concept of cordial labeling [6].There are variety of cordial labeling available in labeling of graphs. Sundaram, Ponraj, and Somasundaram [10] introduced the notion of product cordial labeling. A product cordial labeling of a graph G with vertex set V is a function f from V to {0,1} such that if each edge (uv) is assigned the label f(u)f(v), the number of vertices labeled with 0 and the number of vertices labeled with 1 diff er by at most 1, and the number of edges labeled with 0 and the number of edges labeled with 1 diff er by at most 1. A graph with a product cordial labeling is called a product cordial graph. We use $v_f(0,1)$ = (a, b) to denote the number of vertices with label 1 are a in number and the number of vertices with label 0 are b in number. Similar notion on edges follows for $e_f(0,1)$ = (x, y).

A lot of work is done in this type of labeling so far. One interested in survey may refer Dynamic survey in Graph labeling by J. Gallian.We mention a very short part of it. Sundaram, Ponraj, and Somasundaram have shown that trees; unicyclic graphs of odd order; triangular snakes; dragons; helms; Pm∪Pn; Cm∪Pn; Pm∪K1,n; Wm∪Fn (Fn is the fan Pn+K1); K1,m∪K1,n; Wm∪ K1,n; Wm∪Pn; Wm∪Cn; the total graph of Pn (the total graph of Pn has vertex set V (Pn)∪E(Pn) with two vertices adjacent whenever they are neighbors in Pn); Cn if and only if n is odd; $C_n^{(t)}$, the one-point union of t copies of C_n, provided t is even or both t and n are even; K2+mK1 if and only if m is odd; $C_m \cup P_n$ if and only if m+n is odd; $K_{m,n} \cup P_s$ if s >mn; Cn+2∪K1,n; Kn∪Kn,(n−1)/2 when n is odd; Kn∪Kn−1,n/2 when n is even; and P2 n if and only if n is odd. They also prove that $K_{m,n}$ (m,n> 2), $P_m \times P_n$ (m,n> 2) and wheels are not product cordial and if a (p,q)-graph is product cordial graph, then q 6 (p−1)(p + 1)/4 + 1.In this paper We show that path union of Flag C_3, C_3^+, bull C_3, C_3^{++}etcare families of product cordial graphs.

3. Preliminaries:

3.1 Fusion of vertex. Let G be a (p,q) graph. Let u≠v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges.If u□G_1 and v□G_2, where G_1 is (p_1,q_1) and G_2 is (p_2,q_2) graph. Take a new vertex w and all the edges incident to u and v are joined to w and vertices u and v are deleted. The new graph has p_1+p_2-1 vertices and $q_1 + q_2$ edges. Sometimes this is referred as u is identified with the concept is well elaborated in Clark and Holton[7].

3.2 Crown graph. It is C_n⊙K_2.At each vertex of cycle a n edge was attached. We develop the concept further to obtain crown for any graph. Thus crown (G) is a graph G ⊙K_2.It has a pendent edge attached to each of it's vertex. If G is a (p,q) graph then crown(G) has q+p edges and 2p vertices.

3.3 Flag of a graph G denoted by FL(G) is obtained by taking a graph G=G(p,q).At suitable vertex of G attach a pendent edge.It has p+1 vertices and q+1 edges.

3.4 A bull graph bull(G) was initially defined for a C_3-bull.It has a copy of G with an pendent edge each fused with any two adjacent vertices of G. For G is a (p,q) graph, bull(G) has p+2 vertices and q+2 edges.

3.5 A tail graph (also called as antenna graph) is obtained by fusing a path p_k to some vertex of G. This is denoted by tail(G, P_k). If there are t number of tails of equal length say (k-1) then it is denoted by tail(G, tp_k). If G is a (p,q) graph and a tail P_k is attached to it then tail(G, P_k) has p+k-1 vertices and q+k-1 edges4.

3.6 Path union of G ,i.e.(G) is obtained by taking a path p_m and take m copies of graph G . Then fuse a copy each of G at every vertex of path at given fixed point on G. It has mp vertices and mq +m-1 edges. Where G is a (p,q) graph.

4. Main results:

Theorem 4.1 Path union of $FL(C_3)$ is product cordial iff m is an even number.

Proof: We take a path $P_m = (v_1,e_1, v_2, e_2, ..v_m)$.The copy of $Fl(C_3)$ fused at i^{th} vertex of P_m is given by $(u_{i,1},c_{i,1},u_{i,2},c_{i,2},u_{i,3},c_{i,3},u_{i,4},d_i)$where $u_{i,1}= v_1$ and pendent edge between $v_{i,3}$ and $u_{i,4}$ is $d_i=(u_{i,3}u_{i,4})$.(Note that $c_{i,1}$, $c_{i,2}$, $c_{i,3}$ are cycle edges) Define a function f: V(G) →{0,1} as follows: $f(v_i) = 1$ for i =1, 2, ..x ; $f(v_i) = 0$ for i = x+1, x+2, , …, 2x. $f(u_{i,j}) = 1$ for all i = 1, 2, ..,x and all j = 1, 2, 3, 4. $f(u_{i,j}) = 0$ for all i = x+1, x+2, ..,2x and all j = 1, 2, 3, 4.The label distribution is given by $v_f(0,1)$= (4x,4x,) and $e_f(0,1)$ = (5x,5x-1).Thus the graph is product cordial when m = 2x.

If m is an odd number then the function f will produce $v_f(0) = v_f(1)$ but then $e_f(0)=e_f(1)+2$ and the condition for product cordiality is not satisfied. #

Theorem 4.2 $P_m(\text{bull } C_3)$ is product cordial for al m.

Proof:We take a path $P_m = (v_1,e_1, v_2, e_2, ..v_m)$.The copy of bull($C_3$) fused at i^{th} vertex of P_m is given by cycle C3 given by $(u_{i,1},c_{i,1},u_{i,2},c_{i,2},u_{i,3},c_{i,3},u_{i,1})$ and $\{u_{i,4},u_{i,5}\}$ where $u_{i,1}= v_1$ and the two pendent edges one at $u_{i,1}$are $d_{i,1}=(u_{i,2}u_{i,4})$ and $d_{i,2}=(u_{i,3}u_{i,5})$ at $u_{i,3}$.(note that $c_{i,1}$, $c_{i,2}$, $c_{i,3}$ are cycle edges).

Define a function f: V(G) →{0,1} as follows:

Case m = 2x.

$f(v_i) = 0$ for i =1, 2, ..x ;

$f(v_i) = 1$ for i = x+1, x+2, , …, 2x.

$f(u_{i,j}) = 0$ for all i = 1, 2, ..,x and all j = 1, 2, 3, 4, 5.

$f(u_{i,j}) = 1$ for all i = x+1, x+2, ..,2x and all j = 1, 2, 3, 4, 5.

The label distribution is given by $v_f(0,1)$= (5x,5x,) and $e_f(0,1)$ = (6x,6x-1).Thus the graph is product cordial when m = 2x.

Case m = 2x+1; We follow the labeling for $P_{2x}(\text{bull } C_3)$ part as is given above. for last vertex V_{2x+1} we define f as follows:

i = 2x+1;

$f(u_{i,j}) = 1$ for j = 1, 2, 3,

f(ui,j)= 0 forj= 4, 5.

The label distribution is given by $v_f(0,1)$= (5x+2,5x+3) and $e_f(0,1)$ = (6x+2,6x+3).

Thus the graph is product cordial when m = 2x+1.

Theorem 4.3 $P_m(C_3^+)$ is product cordial for all m.

Proof:We take a path $P_m = (v_1,e_1, v_2, e_2, ..v_m)$.The copy of crown ($C_3^+$) fused at i^{th} vertex of P_m is given by $(u_{i,1},c_{i,1},u_{i,2},c_{i,2},u_{i,3},c_{i,3},u_{i,1})$,$\{u_{i,4},u_{i,5},u_{i,6}\}$ where $u_{i,1}= v_{i,1}$ and the three pendent edges are $(u_{i,j}u_{i,j+3})$for given i= 1, 2, ..mand j = 1,2, 3.(note that $c_{i,1}$, $c_{i,2}$, $c_{i,3}$ are cycle edges). The path union is taken at pendent vertex, as such there are only 2 pendent vertices at i^{th} copy of C_3^+ on $P_m(C_3^+)$.

Define a function f: V(G) →{0,1} as follows:

Case m = 2x.

$f(u_{i,j}) = 0$ for i = 1, 2, ..,x and j= 1, 2, ..,6.

$f(u_{i,j}) = 1$for i = x+1,x+ 2, ,,m and j= 1, 2, ..6.

The label distribution is given by $v_f(0,1)$= (6x,6x,) and $e_f(0,1)$ = (7x,7x-1).

Thus the graph is product cordial when m = 2x.

Case m = 2x+1.

We follow the labeling for $P_{2x}(C_3^+)$ part as is given above. for last vertex V_{2x+1}We define f as follows:

i = 2x+1;

$f(u_{i,j}) = 1$ for j = 1, 2, 3,

f(ui,j)= 0 forj= 4, 5,6.

The label distribution is given by $v_f(0,1)$= (6x+3,6x+3,) and $e_f(0,1)$ = (7x+3,7x+3).

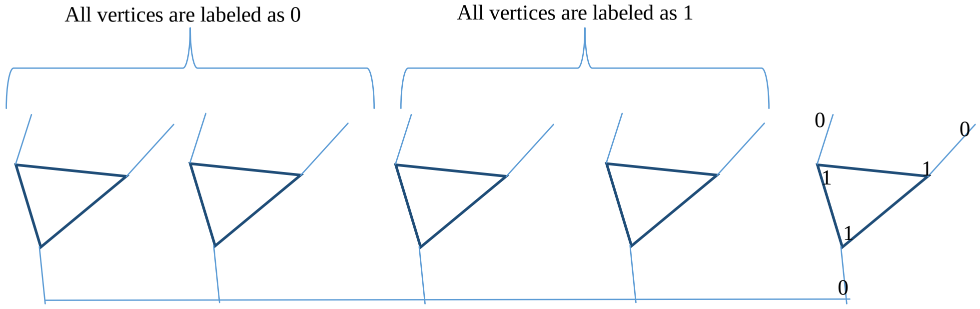

Fig 4.1 : Labeled copy of $P_5(C_3^+)$

Theorem 4.4 Productcordial labeling of Path union on G'=tail(C_3,$2P_2$) given by Pm(G') exists for all m.
Proof: We take a path P_m = (v_1,e_1, v_2, e_2, ..v_m).The copy of tail(C_3,$2P_2$) fused at i^{th} vertex of P_m is given by ($u_{i,1}$,$c_{i,1}$,$u_{i,2}$,$c_{i,2}$,$u_{i,3}$,$c_{i,3}$,$u_{i,1}$) and {$u_{i,4}$,$u_{i,5}$} where $u_{i,1}$= $v_{i,1}$ and the two pendent edges are ($u_{i,j}u_{i,j+k}$); j = 3 and k = 1,2.; given i= 1, 2, ..m. (note that $c_{i,1}$, $c_{i,2}$, $c_{i,3}$ are cycle edges) .
Define a function f: V(G) →{0,1} as follows:
Case m = 2x.

$f(v_i)$ = 0 for i =1, 2, ..x ;
$f(v_i)$ = 1 for i = x+1, x+2, , …, 2x.
$f(u_{i,j})$ = 0 for all i = 1, 2, ..,x and all j = 1, 2, 3, 4, 5.
$f(u_{i,j})$ = 1 for all i = x+1, x+2, ..,2x and all j = 1, 2, 3, 4, 5.

The label distribution is given by $v_f(0,1)$= (5x,5x,) and $e_f(0,1)$ = (6x,6x-1).Thus the graph is product cordial when m = 2x.

Case m = 2x+1
We follow the labeling for P_{2x}(bull C_3) part as is given above. for last vertex V_{2x+1}we define f as follows:
i = 2x+1;

$f(u_{i,j})$ = 1 for j = 1, 2, 3,
f(ui,j)= 0 forj= 4, 5.

The label distribution when m = 2x+1 is given by $v_f(0,1)$= (5x+2,5x+3,) and $e_f(0,1)$ = (6x+3,6x+2).Thus the graph is product cordial when m = 2x+1.
If we change the vertex on tail(C_3,$2P_2$) fused with path P_m There is no change in f as above for m up to 2x. for m = 2x+1 we have to take,i = 2x+1,

$f(u_{i,3})$=1,
$f(u_{i,4})$ = 1;
$f(u_{i,5})$= 1,
$f(u_{i,2})$ =0,
$f(u_{i,1})$ = 0.

The label distribution is also same.In the diagrams below the numbers are vertex labels.

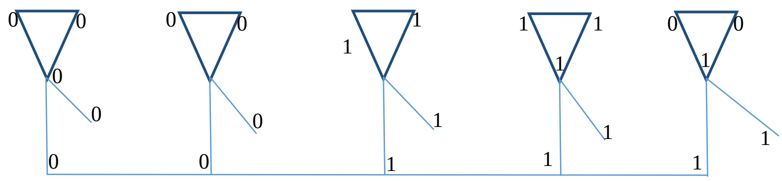

Fig 4.2P_5(tail(C_3,$2P_2$),$v_f(0,1)$= (12,13,) and $e_f(0,1)$ = (15,14)

Thus the graph is product cordial. #

Theorem 4.5 Let G' be a graph obtained from cycle C_3 by attaching two pendent edges each at two adjacent vertices. Then G= P_m(G') is product cordial graph.
Proof: We take a path P_m = (v_1,e_1, v_2, e_2, ..v_m).The copy of G' fused at i^{th} vertex of P_m is given by ($u_{i,1}$,$c_{i,1}$,$u_{i,2}$,$c_{i,2}$,$u_{i,3}$,$c_{i,3}$,$u_{i,1}$),{$u_{i,4}$,$u_{i,5}$,$u_{i,6}$,$u_{i,7}$} where $u_{i,1}$= $v_{i,1}$ and the four pendent edges are ($u_{i,1}u_{i,4}$) and ($u_{i,1}u_{i,5}$)and ($u_{i,2}u_{i,6}$) and ($u_{i,2}u_{i,7}$)i= 1, 2, ..m.(note that $c_{i,1}$, $c_{i,2}$, $c_{i,3}$ are cycle C_3edges).Thus |V(G)| = 7m and |E(G)|= 8m-1

Define a function f: V(G) →{0,1} as follows:
Case m = 2x;

$f(u_{i,j})=0$ for all i = 1, 2, ..,x and j= 1, 2, ..7.
$f(u_{i,j})=1$ for all i = x+1, x+2, ..,2x and j= 1, 2, ..7.
The label distribution is given by $v_f(0,1)= (7x,7x)$ and $e_f(0,1) = (8x,8x-1)$.

Thus the graph is product cordial when m = 2x

Case m = 2x+1;

We follow the labeling for $P_{2x}(G')$ part as is given above. for last vertex V_{2x+1}we define f as follows: i = 2x+1;

$f(u_{i,j})= 1$ for i = 2x+1 and j = 1, 2, 3,4

$f(u_{i,j})=0$ for all i = 2x+1and j= 5, 6, 7;

The label distribution is given by $v_f(0,1)= (7x+3,7x+4,)$ and $e_f(0,1) = (8x+3,8x+4)$.In the diagrams below the numbers are vertex labels.

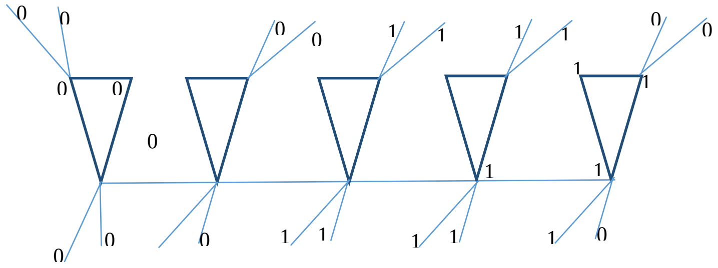

Fig 4.2$P_5(G')$,$v_f(0,1)= (17,18)$ and $e_f(0,1) = (19,20)$

Thus the graph is product cordial. #

Theorem 4.6 let G' stands for a graph obtained from C_3 by attaching two pendent edges at each point on C_3.The path union on G' denoted by $P_m(G')$ is product cordial.

Proof: We take a path $P_m = (v_1,e_1, v_2, e_2, ..v_m)$.The copy of G' fused at i[th] vertex of P_m is given by $(u_{i,1},c_{i,1},u_{i,2},c_{i,2},u_{i,3},c_{i,3},u_{i,1})$ for cycle C_3 and $\{u_{i,4},u_{i,5},u_{i,6},u_{i,7},u_{i,7},u_{i,8}\}$ where $u_{i,1}= v_{i,1}$ and the two pendent edges are $(u_{i,1}u_{i,4})$ and $(u_{i,1}u_{i,5})$ and $(u_{i,2}u_{i,6})$, $(u_{i,2}u_{i,7})$,$(u_{i,3}u_{i,8})$,$(u_{i,3}u_{i,9})$i= 1, 2, ..m. (note that $c_{i,1}$, $c_{i,2}$, $c_{i,3}$ are cycle C_3 edges). The graph has 9m vertices and 10m-1 edges.

Define a function f: V(G) →{0,1} as follows:

Case m = 2x;

$f(u_{i,j}) = 1$ for all i = 1, 2, ..,x and j= 1, 2, ..7.

$f(u_{i,j})=0$ for all i = x+1, x+2, ..,2x and j= 1, 2, ..9.The label distribution is given by $v_f(0,1)= (9x,9x)$ and $e_f(0,1) = (10x,10x-1)$.

Thus the graph is product cordial when m = 2x

Case m = 2x+1;

$f(u_{i,j}) =1$ for all i = 1, 2, ..,x and j= 1, 2, ..9.

$f(u_{i,j})=0$ for all i = x+1, x+2, ..,2x and j= 1, 2, ..9.

$f(u_{i,j})= 1$ for i = 2x+1 and j = 1, 2, ..5

$f(u_{i,j})=0$ for all i = 2x+1 and j= 6,7,8,9;

The label distribution is given by $v_f(0,1)= (9x+4,9x+5,)$ and $e_f(0,1) = (9x+7,9x+6)$.In the diagrams below the numbers are vertex labels.

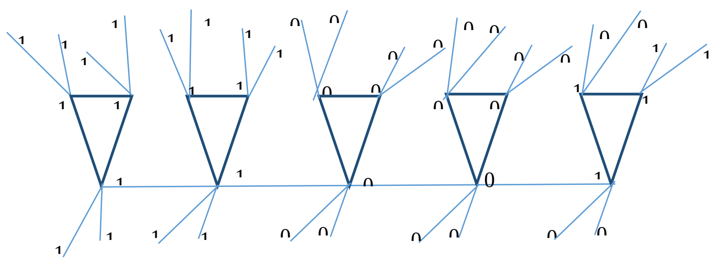

Fig 4.2 $P_5(G')$,$v_f(0,1)= (22,23,)$ and $e_f(0,1) = (25,24)$

Thus the graph is product cordial. #

Conclusions:

In this paper path union of C_3 related graphs are discussed and are shown to be product cordial. We show that following families are product cordial.

1) Path union of $FL(C_3)$ is product cordial iff m is an even number.
2) G' = bull(C_3). Then G = P_m(G') is product cordial for all m.
3) $P_m(C_3^+)$ is product cordial for all m.
4) Product cordial labeling of Path union on G'= tail(C_3,$2P_2$) given by P_m(G') exists for all m.
5) Let G' be a graph obtained from cycle C_3 by attaching two pendent edges each at two adjacent vertices. Then G= P_m(G') is product cordial graph.
6) Then G= P_m(G') is product cordial graph.G' be a graph obtained from cycle C_3 by fusing two pendent vertices each at each vertex of C_3. G= P_m(G') is product cordial on all structures.

Thus it is interesting to study the cycles with pendent edges fused at some or all vertices of it and a path union taken is product cordial (under certain conditions).

In above discussion we have restricted our scope to pendent edges not more than two at any vertex. It is necessary to study such graphs by taking at most t (>2) edges at each vertex and obtain different structures as above.

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