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# One point union cordiality of shel graph

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**Abstract:** In this paper we discuss cordial labeling of shell related graphs.We show that tail($s_4$,$P_2$),$G^{(k)}$ where G = tail($s_4$,$P_3$), tail($s_4$,2-$P_2$) are cordial graphs
**Keywords:** cordial, labeling, shell graph, one point union.
**Subject Classification:** 05C78

## 2. Introduction:

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [6],A dynamic survey of graph labeling by J. Gallian [8] and Douglas West.[9].I. Cahit introduced the concept of cordial labeling[5].f:V(G)→{0,1} be a function. From this label of any edge (uv) is given by |f(u)-f(v)|.Further number of vertices labeled with 0 i.e$v_f(0)$ and the number of vertices labeled with 1 i.e.$v_f(1)$ differ at most by one .Similarly number of edges labeled with 0 i.e. $e_f(0)$ and number of edges labeled with 1 i.e. $e_f(1)$ differ by at most one. Then the function f is called as cordial labeling.Cahit has shown that : every tree is cordial; $K_n$ is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of $C_3$) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel $W_n$ is cordial if and only if n is not congruent to 3 (mod 4).A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian [8].

## 3. Preliminaries:

3.1 Fusion of vertex. Let G be a (p,q) graph. let$u \neq v$ be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges.[9].
3.2 A tail graph (also called as antenna graph) is obtained by fusing a path $p_k$ to some vertex of G. This is denoted by tail(G, $P_k$). If there are t number of tails of equal length say (k-1) then it is denoted by tail(G, $tp_k$). If G is a (p,q) graph and a tail $P_k$ is attached to it then tail(G, $P_k$) has p+k-1 vertices and q+k-1 edges.
3.3 $G^{(K)}$ it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies.If G is a (p, q) graph then $|V(G_{(k)}| = k(p-1)+1$and $|E(G)|= k.q$
3.4 Shell graph $S_n$ is obtained from cycle Cn by taking n-3 concurrent chords from any one vertex on Cn say v to n-3 vertices of Cn which are non-adjacent to v.It has 2n-3 edges and n vertices.

## 4. Theorems proved:

4.1 Theorem.
$G^{(k)}$ is cordial where G = tail($s_4$,$P_2$)
Proof: Define a function f:V(G)→{0,1} as follows. It introduces tho types of labeling units as given below. We combine them suitably to obtain a labeled copy of $G^{(k)}$.There are two cases depending on where the $P_2$ is attached on $S_4$.
Case 1. $P_2$ is attached at one of the two 3-degree vertices of $S_4$.

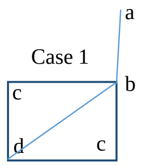

Fig 4.1 Tail($C_4$,$P_2$)

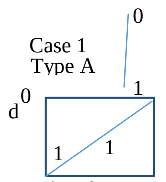

Fig 4.2
$v_f(0,1)=(2,3),e_f(0,1)=(3$

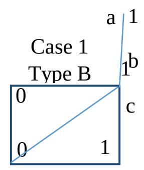

Fig 4.3
$v_f(0,1)=(2,3),e_f(0,1)=(3,$

We can take one point union at vertices 'a', 'b', 'c' or 'd', see fig 4.1, to produce structure1 to structure 4 respectively. All these structures will be pairwise non-isomorphic.

  

In structure 1 type B is used repeatedly .The one point union is taken at vertex 'a' on it for all k of $G^{(k)}$.
In structure 2 type B is used repeatedly .The one point union is taken at vertex 'b' on it for all k of $G^{(k)}$.
In structure 3 type B is used repeatedly .The one point union is taken at vertex 'c' on it for all k of $G^{(k)}$.
In structure 4 type A is used repeatedly .The one point union is taken at vertex 'd' on it for all k of $G^{(k)}$.
The resultant number distribution for all four structures is $v_f(0,1)=(2k,2k+1)$, $e_f(0,1)=(3k,3k)$ for all k.

case 2 : $P_2$ is attached at one of the two 2-degree vertices of $S_4$.

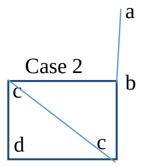


Fig 4.4 Tail($C_4$,$P_2$)

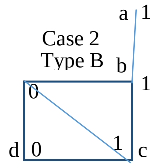


Fig 4.5
$v_f(0,1)=(2,3),e_f(0,1)=(3$

We take one point union on any of the vertices 'a', 'b', 'c' or 'd'( see fig 4.4 above). In that case we get structure1,structure2, structure3 and structure 4 respectively. All these structures will be pairwise non-isomorphic.
In structure 1 type B is used repeatedly .The one point union is taken at vertex 'a' on it for all k of $G^{(k)}$.
In structure 2 type B is used repeatedly .The one point union is taken at vertex 'b' on it for all k of $G^{(k)}$.
In structure 3 type B is used repeatedly .The one point union is taken at vertex 'c' on it for all k of $G^{(k)}$.
In structure 4 type A is used repeatedly .The one point union is taken at vertex 'd' on it for all k of $G^{(k)}$.
The resultant number distribution for all four structures is $v_f(0,1)=(2k,2k+1)$, $e_f(0,1)=(3k,3k)$ for all k. Thus in both cases all structures possible on one point union of tail($S_4$,p2) are cordial.

#4.2 Theorem.
$G^{(k)}$ is cordial where G = tail($s_4$,$P_3$)
Proof: Define a function f:V(G)→{0,1} as follows. It introduces ffive types of labeling units as given below. We combine them suitably to obtain a labeled copy of $G^{(k)}$.There are two cases depending on where the $p_2$ is attached on $S_4$.

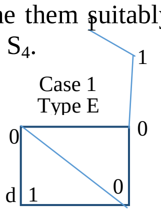


Fig 4.10 $v_f(0,1)=$
$(3,3),e_f(0,1)=(4,3)$

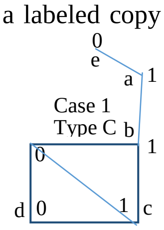


Fig 4.11 $v_f(0,1)=$
$(3,3),e_f(0,1)=(3,4)$

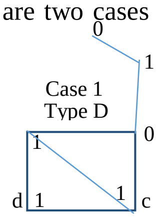


Fig 4.12 $v_f(0,1)=$
$(2,4),e_f(0,1)=(3,4)$

We take one point union on any of the vertices 'e', 'a', 'b', 'c' or 'd'( see fig 4.7 above).
In that case we get structure1,structure2, structure3, structure 4 and structure 5 respectively.
All these structures will be pairwise non-isomorphic.
In structure 1, type C and type A are used repeatedly.
The one point union is taken at vertex 'e' on it. Type C is used as $i^{th}$ copy if i≡1 (mod4) and copy A is used at $i^{th}$ copy if i≡ 0 (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.
In structure 2, type C and type B are used repeatedly .T
he one point union is taken at vertex 'a' on it. Type C is used as $i^{th}$ copy if i≡1 (mod 2) and copy B is used at $i^{th}$ copy if i≡ 0 (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.
In structure 3, type C and type B are used repeatedly .The one point union is taken at vertex 'b' on it.
Type C is used as $i^{th}$ copy if i≡1 (mod4) and copy B is used at $i^{th}$ copy if i≡ 0 (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.
In structure 4, type C and type B are used repeatedly .
The one point union is taken at vertex 'c' on it. Type C is used as $i^{th}$ copy if i≡1 (mod4) and copy B is used at $i^{th}$ copy if i≡ 0 (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.
In structure 5, type E and type D are used repeatedly .
The one point union is taken at vertex 'd' on it. Type E is used as $i^{th}$ copy if i≡1 (mod4) and copy D is used at $i^{th}$ copy if i≡ 0 (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.

  

The label number distribution for structure 1 is as follows:On vertices $v_f(0,1)=(3+5x,3+5x)$ and on edges $e_f(0,1)=(3+7x,4+7x)$ if kis of type k=2x+1, x =0,1,2..And if k=2x,x=1,2,..we have on vertices $v_f(0,1)=(1+5x,5x)$ and on edges $e_f(0,1)=(7x,7x)$if kis of type k=2x+1, x =0,1,2..

The label distribution for structure 2, structure 3 and structure 4 is as follows:On vertices $v_f(0,1)=(3+5x,3+5x)$ and on edges $e_f(0,1)=(3+7x,4+7x)$ if kis of type k=2x+1, x =0,1,2..And if k=2x,x=1,2,..we have on vertices $v_f(0,1)=(5x,5x+1)$ and on edges $e_f(0,1)=(7x,7x)$if k is of type k=2x+1, x =0,1,2..

The label distribution for structure 5 is as follows:On vertices $v_f(0,1)=(3+5x,3+5x)$ and on edges $e_f(0,1)=(4+7x,3+7x)$ if kis of type k=2x+1, x =0,1,2..And if k=2x,x=1,2,..we have on vertices $v_f(0,1)=(5x,5x+1)$ and on edges $e_f(0,1)=(7x,7x)$if k is of type k=2x+1, x =0,1,2..

Case 2. $P_2$ is attached at one of the two 3-degree vertices of $S_4$.

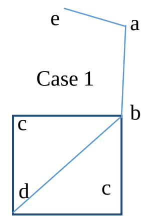


Fig 4.13 tail($C_4$,$P_2$)

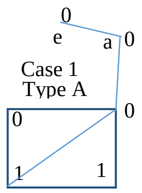


Fig 4.14

$v_f(0,1)=(4,2),e_f(0,1)=(4$

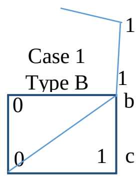


Fig 4.15

$v_f(0,1)=(2,4),e_f(0,1)=(4,$

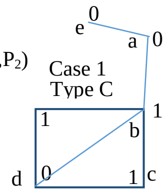


Fig 4.16

$v_f(0,1)=(3,3),e_f(0,1)=(3$

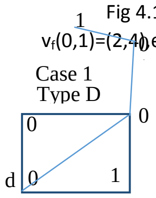


Fig 4.17

$v_f(0,1)=(4,2),e_f(0,1)=(4$

We can take one point union at vertices ‘e’, ‘a’, ‘b’, ‘c’ or ‘d’, see fig 4.13, to produce structure 1 to structure 5 respectively. All these structures will be pairwise non-isomorphic.

In structure 1, type C and type A are used repeatedly .The one point union is taken at vertex ‘e’ on it.Type C is used as $i^{th}$ copy if i≡1 (mod4) and copy A is used at $i^{th}$ copy if i≡ 0 (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.

In structure 2, type C and type A are used repeatedly .The one point union is taken at vertex ‘a’ on it.Type C is used as $i^{th}$ copy if i≡1 (mod 2) and copy A is used at $i^{th}$ copy if i≡ 0 (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.

In structure 3, type C and type B are used repeatedly .The one point union is taken at vertex ‘b’ on it.Type C is used as $i^{th}$ copy if i≡1 (mod4) and copy B is used at $i^{th}$ copy if i≡ 0 (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.

In structure 4, type C and type B are used repeatedly .The one point union is taken at vertex ‘c’ on it.Type C is used as $i^{th}$ copy if i≡1 (mod4) and copy B is used at $i^{th}$ copy if i≡ 0 (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.

In structure 5, type C and type D are used repeatedly .The one point union is taken at vertex ‘d’ on it.Type C is used as $i^{th}$ copy if i≡1 (mod4) and copy D is used at $i^{th}$ copy if i≡ 0 (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.

The label number distribution for structure 1, structure 2 and structure 5 is as follows:On vertices $v_f(0,1)=(3+5x,3+5x)$ and on edges $e_f(0,1)=(3+7x,4+7x)$ if kis of type k=2x+1, x =0,1,2..And if k=2x,x=1,2,..we have on vertices $v_f(0,1)=(1+5x,5x)$ and on edges $e_f(0,1)=(7x,7x)$if kis of type k=2x+1, x =0,1,2..

The label number distribution for structure 3 and structure 4 is as follows:On vertices $v_f(0,1)=(3+5x,3+5x)$ and on edges $e_f(0,1)=(3+7x,4+7x)$ if kis of type k=2x+1, x =0,1,2..And if k=2x,x=1,2,..we have on vertices $v_f(0,1)=(5x,5x+1)$ and on edges $e_f(0,1)=(7x,7x)$if kis of type k=2x+1, x =0,1,2..

Thus the graph is cordial. #

4.3 Theorem.

$G^{(k)}$ is cordial where G = tail($s_4$,2-$P_2$)

Proof: Define a function f:V(G)→{0,1} as follows. It introduces four types of labeling units as given below. Type A and Type B are not cordial while Type C and type D have same label number but different distribution.

We combine them suitably to obtain a labeled copy of $G^{(k)}$.There are two cases depending on where the two copies of $p_2$are attached on $S_4$.

Case 1 : both $p_2$ are attached at 2-degree vertex of $S_4$.

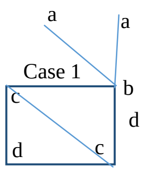


Fig 4.18 Tail($C_4$,$P_2$)

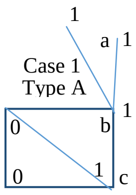


Fig 4.19 $v_f(0,1)=(2,4), e_f(0,1)=(4,3)$

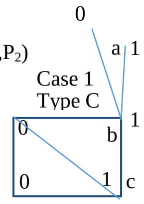


Fig 4.21 $v_f(0,1)=(3,3), e_f(0,1)=(3,4)$

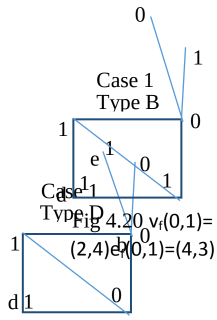


Fig 4.20 $v_f(0,1)=(2,4), e_f(0,1)=(4,3)$

Fig 4.22 $v_f(0,1)=(3,3), e_f(0,1)=(3,4)$

We take one point union on any of the vertices 'a', 'b', 'c' or 'd'( see fig 4.18 above). In that case we get structure1,structure2, structure3 and structure 4 respectively. All these structures are pairwise non-isomorphic.
In structure 1, type C and type A are used repeatedly .The one point union is taken at vertex 'a' on it. Type C is used as $i^{th}$ copy if $i \equiv 1$ (mod4) and copy A is used at $i^{th}$ copy if $i \equiv 0$ (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.
In structure 2, type C and type A are used repeatedly .The one point union is taken at vertex 'b' on it.Type C is used as $i^{th}$ copy if $i \equiv 1$ (mod 2) and copy A is used at $i^{th}$ copy if $i \equiv 0$ (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.
In structure 3, type C and type A are used repeatedly .The one point union is taken at vertex 'c' on it.Type C is used as $i^{th}$ copy if $i \equiv 1$ (mod4) and copy A is used at $i^{th}$ copy if $i \equiv 0$ (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.
In structure 4, type D and type B are used repeatedly .The one point union is taken at vertex 'd' on it.Type D is used as $i^{th}$ copy if $i \equiv 1$ (mod4) and copy B is used at $i^{th}$ copy if $i \equiv 0$ (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$
The label number distribution is on vertices $v_f(0,1)=(3+5x,3+5x)$ and on edges $e_f(0,1)=(3+7x,4+7x)$when k = 2x, x= 0, 1, 2.. and when k is of type k = 2x, on vertices we have $v_f(0,1)=(5x, 1+5x)$ and on edges $e_f(0,1)=(7x,7x)$.

Case 2. Two $P_2$paths are attached at one of the two 3-degree vertices of $S_4$.

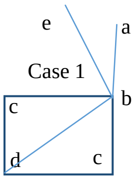


Fig 4.6 Tail($C_4$,$P_2$)

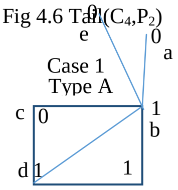


Fig 4.7 $v_f(0,1)=(3,3), e_f(0,1)=(3$

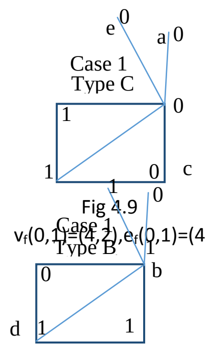


Fig 4.9 $v_f(0,1)=(4,2), e_f(0,1)=(4$

Fig 4.8 $v_f(0,1)=(2,4), e_f(0,1)=(4,$

can take one point union at vertices 'e', , 'b', 'c' or 'd', see fig 4.6, to produce structure 1 to structure 4 respectively. All these structures will be pairwise non-isomorphic.
In structure 1, type A and type C are used repeatedly .The one point union is taken at vertex 'e' on it.Type A is used as $i^{th}$ copy if $i \equiv 1$ (mod4) and copy C is used at $i^{th}$ copy if $i \equiv 0$ (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.

  

In structure 2, type A and typeB are used repeatedly .The one point union is taken at vertex 'b' on it.Type A is used as $i^{th}$ copy if i≡1 (mod 2) and copy B is used at $i^{th}$ copy if i≡ 0 (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.
In structure 3, type A and type B are used repeatedly .The one point union is taken at vertex 'd' on it.Type A is used as $i^{th}$ copy if i≡1 (mod4) and copy B is used at $i^{th}$ copy if i≡ 0 (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.
In structure 4, type A and type C are used repeatedly .The one point union is taken at vertex 'c' on it.Type A is used as $i^{th}$ copy if i≡1 (mod4) and copy C is used at $i^{th}$ copy if i≡ 0 (mod 2) for all i= 1,2, .k, in construction of $G^{(k)}$.
The label distribution for structure 1 and structure 4 is as follows:On vertices $v_f(0,1)=(3+5x,3+5x)$ and on edges $e_f(0,1)=(3+7x,4+7x)$ if kis of type k=2x+1, x =0,1,2..And if k=2x,x=1,2,..we have on vertices $v_f(0,1)=(1+5x,5x)$ and on edges $e_f(0,1)=(7x,7x)$if kis of type k=2x+1, x =0,1,2..

The label distribution for structure 2 and structure 3 is as follows:On vertices $v_f(0,1)=(3+5x,3+5x)$ and on edges $e_f(0,1)=(3+7x,4+7x)$ if kis of type k=2x+1, x =0,1,2..And if k=2x,x=1,2,..we have on vertices $v_f(0,1)=( 5x,5x+1)$ and on edges $e_f(0,1)=(7x,7x)$ if kis of type k=2x+1, x =0,1,2..

Thus the graph is cordial. #
Conclusions:We show that tail($s_4$,$P_2$),$G^{(k)}$ where G = tail($s_4$,$P_3$), tail($s_4$,2-$P_2$)
are cordial graphs.

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