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Using the Optimal Design for the Vehicle Dynamics to Achieve Maximum Tire's Friction Force in Rear Axle

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Abstract : In this paper, creating the optimization design to find the vehicle's parameters such as car's speed, v; displacement from front-end to focus point, a; the height of focus point h_g and the acceleration of the car's body at the maximum rear axle tire's friction force, j. Beside, getting the maximum tire's friction force has a major profit in vehicle motive characteristic. It improves the quality of the braking and upgrades the drag feature of the car. We use the Matlab software and Excel Slove to calculator the parameter of the car base on optimal algorithm. Results received on two method which are original nonlinear and linearization with Simplex method depend on initial condition. Parameters after optimization can be actual application on real car for comparison and to find the best results.

Keywords: optimization, friction of tire, maximum friction force, Linearization, Optimum Design

I. INTRODUCTION

Designer is interested in improving the moving characteristics of vehicle because it is very important in vehicle dynamics. It involves in moving car safety and acceleration ability. I am concerned about how to optimize the car's factors that affect to the tire's friction force. So, that is the reason why I chose this topic.

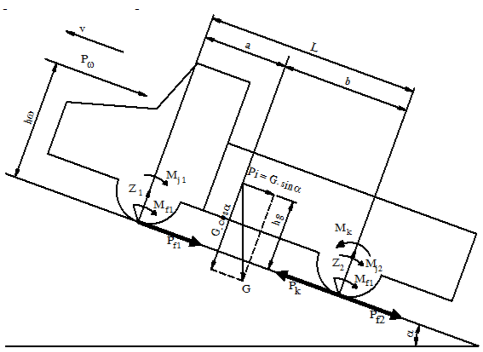

Figure1. The simplify car's model [1]

Where:

P_w- is the wind force affected to the car's body

P_j- is the acceleration force

P_f - is the rolling drag force affected to moving car

Z_1; Z_2- are the vertical forces affected to the front and rear axle

α - is the slope of the road

$h_w \sim = h_g$ - are the height of wind force and the height of center point of the car's weight

The optimal value is the maximum of the tire's force friction with the constraints:

- The car's speed must be positive, $10 \leq v \leq 70$
- The vertical road contrast force to the front axle must be greater or equal to zero to prevent the car not be rotated, $Z_1 \geq 0$;
- Total resistant forces is equal or less than the friction force to enable the car moving forward, $\sum P_i \leq P_\varphi$;
- The ratio between the a size and the length L is: $1/4 \leq a/L \leq 3/4$;
- The ratio between the height of focus point h_g and the highest size H is, $1/5 \leq h_g/H \leq 1/2$;
- The acceleration of the car body must be: $-g \leq j \leq g$.

II. APPROACH AND METHODS

We use the optimum design to solve the problem. First, we use the option data-solve in excel to find out the optimal points and optimal value. Second, we linearize problem and use the simplex method to solve it. We use the option data-solve in excel again to find out the optimal of linearization problem. After that, we will use the algorithms in Mat-lab to solve the original nonlinear problem. The last, we compare the results and made conclusion.

• Step in detail:

Following five steps to define OPT problem:

Step 1: Project statement

• Achieve maximum rear-axle friction force $F_{2\varphi}$

• Find the optimal design of vehicle's parameters: v, a, h_g and j.

Step 2: Data and information collection

•The rear axle friction force $F_{2\varphi} = Z_2\varphi_2$

• The front axle vertical force

$$Z_1 = \frac{Gcos\alpha(b - fr_b) - (Gsin\alpha + P_j + P_w)h_g}{L}$$

• The rear axle vertical force

$$Z_2 = \frac{Gcos\alpha(a + fr_b) + (Gsin\alpha + P_j + P_w)h_g}{L}$$

• The friction coefficient of the rear tire depends on the load: $\varphi_2 = 0.75 - \frac{Z_2/2-2200}{116000}$

• The accelerate force: $P_j = (1.04 + 0.05i_h^2)\frac{G}{g}j$

• The drag force: $P_w = \frac{C_D A\rho(v+v_w)^2}{2}$

• The total resistant forces:

$$\sum P_i = P_f + P_\alpha + P_j + P_w = fGcos\alpha + Gsin\alpha + (1.04 + 0.05i_h^2)\frac{G}{g}j + \frac{C_D A\rho(v + v_w)^2}{2}$$

• The total car's friction force: $P_\varphi = G\varphi = G(0.75 - \frac{G/4-2200}{116000})$

Step 3: Identification of Design variables

We have four design variables: v, a, h_g and j.

Step 4: Identification of a criterion to be optimized [2]

Objective function: max $F_{2\varphi}(v, a, h_g, j)$

$$F_{2\varphi}(v,a,h_g,j) = Z_2\left(0.75 - \frac{Z_2/2 - 2200}{116000}\right) = Z_2\left(0.769 - \frac{Z_2}{232000}\right)$$

$$= 0.769\left[\frac{Gcos\alpha(a + fr_b) + (Gsin\alpha + P_j + P_w)h_g}{L}\right] - \frac{\left[\frac{Gcos\alpha\ (a+fr_b)+(Gsin\alpha\ +P_j+P_w)h_g}{L}\right]^2}{232000}$$

Step 5: Identification of constraints

• $G_1 = Z_1 = \frac{Gcos\alpha\ (b-fr_b)-(Gsin\alpha\ +P_j+P_w)h_g}{L} \geq 0$

• $G_2 = \sum P_i = fGcos\alpha + Gsin\alpha + (1.04 + 0.05i_h^2)\frac{G}{g}j + \frac{C_D A\rho(v+v_w)^2}{2} \leq P_\varphi = G(0.75 - \frac{G/4-2200}{116000})$

• G_3=a-L/4 ≥0

• G_4= a-3L/4≤ 0

• G_5=h_g-H/5 ≥0

• G_6= h_g-H/2≤ 0

• G_7= j +g ≥0

• G_8= j-g≤ 0

• G_9= v≥0

• G_{10}= v-70≤0

Table 1: The table of initial condition (base line): Mer CL63

Value	Characteristic	Unit
G=20464	Total car's weight	N
f=0.015	Rolling resistant coefficient	-
$\alpha = 3^0$	The road angle	Degree
i_h =2.5	The transmission ratio	-
g=9.81	The gravity accelerate	m/s^2
r_b=0.254	Tire radius	m
a=0.9	Distance between the front-end point and the mass center point	m
b=1	Distance between the rear-end point and mass center point	m
L=2.0	Total car's length	m
h_g=0.215	The height of mass center point	m
H=0.558	The car's height	m
C_D=0.35	Drag coefficient	-
A=0.907	The front drag area	m^2
ρ=1.225	The air density	kg/m^3
v=20	The car's speed	m/s
v_w=2	The wind speed	m/s
j=0	The car's body acceleration	m/s^2

The problem will become:

Given

+ Design variables: v, a, h_g, j [2],[3]

+ Object to:

$\min f(v,a,h_g,j)=4.31(10^{-6})\{10218(a+0.0038)+[535+1410j+0.0972(v+2)^2]h_g\}^2-0.769\{10218(a+0.0038)+[535+1410j +0.0972(v+2)^2]h_g\}$

+ Subject to:

- $g_1=10218(a+0.0038)+[535+1410j+0.0972(v+2)]h_g-20434\leq 0$
- $g_2= 306.6a +2821j+0.1944(v+2)^2-13763\leq 0$
- $g_3=0.5 - a \leq 0$
- $g_4= a - 1.5\leq 0$
- $g_5=0.1116 - h_g \leq 0$
- $g_6= h_g - 0.279\leq 0$
- $g_7= -j - 9.81 \leq 0$
- $g_8= j-9.81\leq 0$
- $g_9= 10-v\leq 0$
- $g_{10}= v-70\leq 0$

Table 2: The results of standard optimal

Answer	v*	a*	hg*	j*	f*	Fφ2*
Optimal Values	70	1.5	0.28	4.3585	-12057	12057
Units	m/s	m	m	m/s^2	N	N

Using Optimality Conditions for Linearization Constrained Problems:

- **Linearize the problem [2],[4]:**

+ Linearization the cost function f and two constrains g_1 and g_2 with the initial conditions are: v_0= 20; a_0=0.9; h_{g0}=0.215; j_0=0.

$$\frac{\partial f}{\partial v}=1.68(10^{-6})\{10218(a+0.0038)+[535+1410j+0.0972(v+2)^2]h_g\}(v+2)h_g - 0.1495(v+2)h_g$$

$$\frac{\partial f}{\partial a}=0.088\{10218(a+0.0038)+[535+1410j+0.0972(v+2)^2]h_g\}-7857$$

$$\frac{\partial f}{\partial hg}=8.62(10^{-6})\{10218(a+0.0038)+[535+1410j+0.0972(v+2)^2]h_g\}[535+1410j+0.0972(v+2)^2] - [411.4+1084j+0.0747(v+2)^2]$$

$$\frac{\partial f}{\partial j}=0.012\{10218(a+0.0038)+[535+1410j+0.0972(v+2)^2]h_g\}h_g-1084h_g$$

$$f=f(v_0,a_0,h_0,j_0)+\frac{\partial f}{\partial v}(v_0,a_0,h_0,j_0)(v-v_0)+\frac{\partial f}{\partial a}(v_0,a_0,h_0,j_0)(a-a_0)+\frac{\partial f}{\partial hg}(v_0,a_0,h_0,j_0)(h_g-h_{g0})+\frac{\partial f}{\partial j}(v_0,a_0,h_0,j_0)(j-j_0) = -6820.4-0.6328(v-20)-7033.3(a-0.9)-437(h_g-0.215)-209(j-0)=-383.8-0.6328v-7033.3a-437h_g-209j$$

$$\frac{\partial g1}{\partial v}=0.1944(v+2)h_g$$

$$\frac{\partial g1}{\partial a}=10218$$

$$\frac{\partial g1}{\partial hg}=535+1410j+0.0972(v+2)^2$$

$$\frac{\partial g1}{\partial j}=1410h_g$$

$$g1=g1(v_0,a_0,h_{g0},j_0)+\frac{\partial g1}{\partial v}(v_0,a_0,h_0,j_0)(v-v_0)+\frac{\partial g1}{\partial a}(v_0,a_0,h_0,j_0)(a-a_0)+\frac{\partial g1}{\partial hg}(v_0,a_0,h_0,j_0)(h_g-h_{g0})+\frac{\partial g1}{\partial j}(v_0,a_0,h_0,j_0)(j-j_0)=-11074+0.9195(v-20)+10218(a-0.9)+582(h_g-0.215)+303.2(j-0)=-20414+0.9195v+10218a+582h_g+303.2j$$

$$\frac{\partial g2}{\partial v}=0.3889(v+2)$$

$$\frac{\partial g2}{\partial a}=306.6$$

$$\frac{\partial g2}{\partial hg}=0$$

$$\frac{\partial g2}{\partial j}=2821$$

$$g^2=g^2(v_0,a_0,h_0,j_0)+\frac{\partial g2}{\partial v}(v_0,a_0,h_0,j_0)(v-v_0)+\frac{\partial g2}{\partial a}(v_0,a_0,h_0,j_0)(a-a_0)+\frac{\partial g2}{\partial hg}(v_0,a_0,h_0,j_0)(h_g-h_{g0})+\frac{\partial g2}{\partial j}(v_0,a_0,h_0,j_0)(j-j_0)=$$

$$=-13393+8.5558(v-20)+306.6(a-0.9)+0(h_g-0.215)+2821(j-0)=-13840+8.5558v+306.6a+2821j$$

- $f=-383.8-0.6328v-7033.3a-437h_g-209j$
- $g_1= -20414+0.9195v+10218a+582h_g+303.2j\leq 0$
- $g_2=-13840+8.5558v+306.6a+2821j\leq 0$

- **Transform to Standard LP Unrestricted Variables:**

Object to: min $f=-383.8-0.6328v-7033.3a-437h_g-209j$

Subject to:

- $g_1= 0.9195v+10218a+582h_g+303.2j\leq 20414$
- $g_2=8.5558v+306.6a+2821j\leq 13840$
- $g_3= a \geq 0.5$
- $g_4= a \leq 1.5$
- $g_5=h_g \geq 0.1116$
- $g_6= h_g \leq 0.279$
- $g_7= j_2-j_1\leq 9.81$

- $g_8 = j_1 - j_2 \le 9.81$
- $g_9 = v \ge 10$
- $g_{10} = v \le 70$

Let $x_1=v$

$x_2=a$
$x_3=h_g$
$x_4=j_1$
$x_5=j_2$

- **Solve problem using simplex method [2]:**

Transform to Standard LP for Simplex Method:
Subject to:

- $0.9195x_1+10218x_2+582x_3+303.2x_4-303.2x_5+s_1=20414$ — s_1 is slack, let $x_6=s_1$
- $8.5558x_1+306.6x_2+2821x_4-2821x_5+s_2=13840$ — s_2 is slack, let $x_7=s_2$
- $x_2-s_3+s_4=0.5$ — s_3 is surplus, let $x_8=s_3$ and s_4 is artificial, let $x_{16}=s_4$
- $x_2+s_5= 1.5$ — s_5 is slack, let $x_9=s_5$
- $x_3-s_6+s_7= 0.1116$ — s_6 is surplus, let $x_{10}=s_6$ and s_7 is artificial, let $x_{17}=s_7$
- $x_3+s_8= 0.279$ — s_8 is slack, let $x_{11}=s_8$
- $x_5-x_4+s_9=9.81$ — s_9 is slack, let $x_{12}=s_9$
- $x_4-x_5+s_{10}=9.81$ — s_{10} is slack, let $x_{13}=s_{10}$
- $x_1-s_{11}+s_{12}=10$ — s_{11} is surplus, let $x_{14}=s_{11}$ and s_{12} is artificial, let $x_{18}=s_{12}$
- $x_1+s_{13}=70$ — s_{13} is slack, let $x_{15}=s_{13}$

Where: $x_1, x_2, \ldots, x_{18} \ge 0$

III. RESULT AND DISCUSSION

Table 3: Results for the original problem:

Method	Original	v*	a*	hg*	j*	f*	Fφ2*
Excel-Solve	**Optimal Values**	70	1.5	0.279	4.3585	-12057	12057
Matlab	**Optimal Values**	70	1.5	0.279	4.3585	-12057	12057
	Units	m/s	m	m	m/s^2	N	N

Table 4: Results for the linearization problem:

Method	Linearization	v*	a*	hg*	j*	f*	Fφ2*
Excel-Solve	**Optimal Values**	10	1.5	0.279	4.7127	-12047	12047
Simplex	**Optimal Values**	10	1.5	0.279	4.7127	-12047	12047
	Units	m/s	m	m	m/s^2	N	N

We can see the results on the tables and easily to realize the final goals. All optimal variables point and friction forces with the same problem are the same. However, in the original problem the maximum friction force is **Fφ_2**=12057 N and in the linearization problem the maximum friction force is **Fφ_2**=12047 N. The difference value, just 10N, is very small.

We can see the car's speed of two problems is not the same, 70m/s in original and 10m/s in linearization. And the acceleration is a difference too, 4.3585 m/s2 in the original and 4.7127 m/s2 in the linearization. Because we linearize the problem with one base line, the initial condition of the car, we get the difference of speed, acceleration and maximum friction force.

On the whole, The original nonlinear OPT and linearization OPT method results get the cost function different, but the results of linearization which is acceptable for real car.

IV. CONCLUSION

From the result we see that:

- ✓ The OPT points alwayslie on the boundary. At each OPT step, at least four constrains are active
- ✓ The result of the linearization problem, simplex method, depends on the base line (initial condition). We have to do more with some new base lines to get more accuracy
- ✓ From the results, we can use these parameters to do some real experiments
- ✓ With multivariable functions problem, using OPT solution will decrease the experiment time and lower the cost.
- ✓ With the results of OPT, we Change some parameters of the experimental vehicle to do some experiments
- ✓ Using the results got from experiments to do OPT again to achieve the best goals
- ✓ Applied in the real model.

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