

More parts prime numbers

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Abstract: After defining the sets, more parts prime numbers will be presented, the four-parts prime numbers from 2237 to 11020507, the five-parts prime numbers from 22273 to 77773, the six-parts prime numbers from 222323 to 373777. How many four-parts prime numbers are there in the interval $(10^{n-1}, 10^n)$, where n is a positive integer number and $n \geq 4$? On the one hand, it has been counted by computer with up to 28-digits. On the other hand, the function (1) gives the approximate number of the four-parts prime numbers in the interval $(10^{n-1}, 10^n)$.

1. Introduction

The sets of special prime numbers within the set of prime numbers are well-known. For instance, left-truncatable primes (If we leave the initial digits out, the remainder will be prime.), right-truncatable primes (If we leave the last digits out, the remainder will be prime.), the Erdős-primes (the sum of the digits is prime) [8], Fibonacci-primes ($F_0=0, F_1=1, F_n=F_{n-1}+F_{n-2}$), Gauss-primes (in the form $4n+3$), Leyland-primes (in the form x^y+y^x , where $1 \leq x \leq y$), Pell-primes ($P_0=0, P_1=1, P_n=2P_{n-1}+P_{n-2}$), Bölcsföldi-Birkás-Ferenczi primes (all digits are prime and the number of digits is prime), Bölcsföldi-Birkás prime numbers (all digits are prime, the number of digits is prime, the sum of digits is prime), etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found further sets of special prime numbers within the set of prime numbers. These are the sets of more parts prime numbers.

2. Four- parts prime numbers [3], [9], [10], [11], [12].

Definition: a positive integer number is four-parts prime number, if

a/ the positive integer number is prime,

b/ the number of digits is $4k$, where $k \geq 1$ integer,

c/ dividing it into four equally long parts, every part is a prime.

The four-parts prime numbers are as follows (the last digit can only be 3 or 7):

2237, 2273, 2333, 2357, 2377, 22557, 2757, 2777, 3253, 3257, 3323, 3373, 3527, 3533, 3557, 3727, 3733, 5227, 5233, 5237,
 5273, 5323, 5333, 5527, 5557, 5573, 5737, 7237, 7253, 7333, 7523, 7537, 7573, 7577, 7723, 7727, 7753, 7757, 11020213, 11020241, 11020253, 11020271, 11020297, 11020313, 11020397, 11020507, etc.

Four-parts prime number „ p ” has the following sum form:

$$p = \sum_{j=0}^{k(p)} e_j(p) \cdot 10^j \quad \text{where } e_j(p) \in \{2, 3, 5, 7\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) \in \{3, 7\} \text{ and } \sum_{j=0}^{k(p)} e_j(p) \text{ is prime.}$$

$K(n)$ is the factual frequency of four-parts prime numbers in the interval $(10^{n-1}, 10^n)$, where $n \geq 4$ integer.

$K(4)=38, K(8)=41336$.

$L(n)$ is the factual frequency of four-parts prime numbers in the interval $(10^{n-1}, 10^n)$, where all digits are 3 or 7.

$L(4)=3, L(8)=3, L(12)=31, L(16)=5, L(20)=0, L(24)=420, L(28)=1923$.

M(n) funktion gives the number of four-parts prime numbers in the interval $(10^{n-1}, 10^n)$, where all digits are 3 or 7.

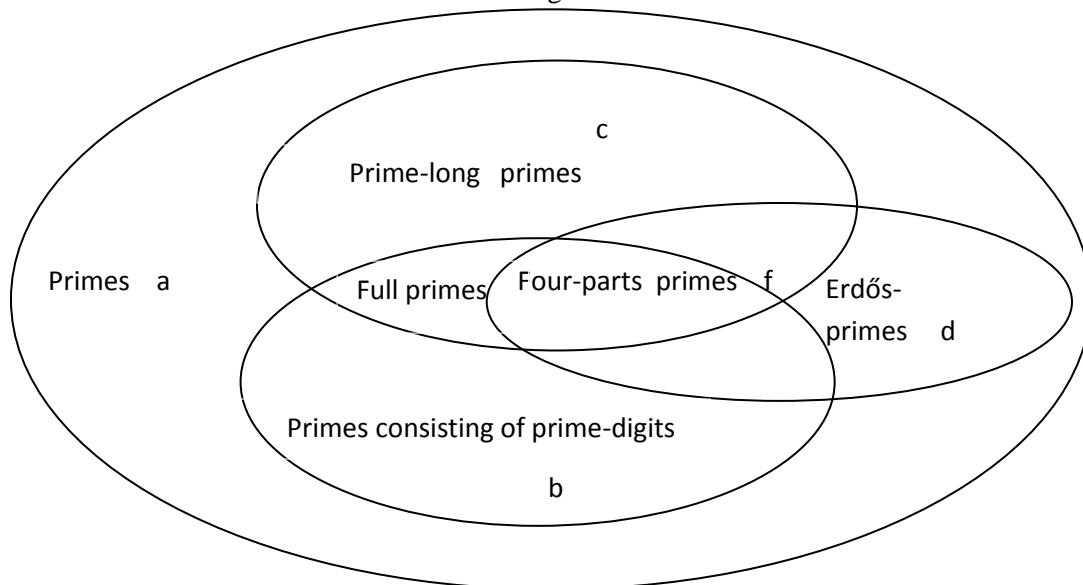
We think that

$$M(n)=1,3355^{n-2} \quad \text{where } n \geq 4 \text{ integer.} \quad (1)$$

The L(n) factual number of four-parts prime numbers and the number of four-parts prime numbers calculated according to funktion M(n) (where all digits are 3 or 7) are as follows:

Number of digits	The L(n) factual number in the interval $(10^{n-1}, 10^n)$	The number calculated according to funktion $M(n)=1,3355^{n-2}$	L(n)/M(n)
n	L(n)	M(n)	
4	3	1,78	1,69
8	3	5,67	0,53
12	31	18,05	1,72
16	5	57,41	0,09
20	0	182,64	0
24	420	580,99	0,72
28	1923	1848,17	1,04

Fig.1



3. Five-parts prime numbers [3], [9], [10], [11], [12].

Definition: a positive integer number is five-parts prime number, if

- a/ the positive integer number is prime,
- b/ the number of digits is $5k$, where $k \geq 1$ integer,
- c/ dividing it into five equally long parts, every part is a prime.

The five-parts prime numbers are as follows (the last digit can only be 3 or 7):

{22273, 22277, 22573, 22727, 22777, 23227, 23327, 23333, 23357, \ 23537, 23557, 23753, 23773, 25237, 25253, 25357, 25373, 25523, 25537, \ 25577, 25733, 27253, 27277, 27337, 27527, 27733, 27737, 27773, 32233, \ 32237, 32257, 32323, 32327, 32353, 32377, 32533, 32537, 32573, 33223, \ 33353, 33377, 33533, 33577, 33757, 33773, 35227, 35257, 35323, 35327, \ 35353, 35527, 35533, 35537, 35573, 35753, 37223, 37253, 37273, 37277, \ 37337, 37357, 37537, 37573, 52223, 52237, 52253, 52553, 52727, 52733, \ 52757, 53233, 53323, 53327, 53353, 53377, 53527, 53773, 53777, 55333, \

55337, 55373, 55733, 57223, 57373, 57527, 57557, 57727, 57737, 57773, \
 72223, 72227, 72253, 72277, 72337, 72353, 72533, 72577, 72727, 72733, \
 73237, 73277, 73327, 73523, 73553, 73727, 73757, 75223, 75227, 75253, \
 75277, 75323, 75337, 75353, 75377, 75527, 75533, 75553, 75557, 75577, \
 75773, 77,237, 77323, 77377, 77527, 77557, 77573, 77723, 77773}

{1102020253, 1102020259, 1102020289, 1102020307, 1102020313, \
 1102020319, 1102020389, 1102020397, 1102020511, 1102020541, \
 1102020559, 1102020707, 1102020743, 1102020761, 1102021111, \
 1102021153,

F(5)=128
 F(10)= 864741

V(n) is the factual number of five-parts prime numbers in the interval $(10^{n-1}, 10^n)$, where all digits are 3 or 7.

V(5)=5 {33377, 33773, 37337, 77377, 77773}
 V(10)= 4, {3737733773, 3773733737, 7337377373, 7337737373}
 V(15)= 77
 V(20)= 23
 V(25)= 164
 V(30)= 3898
 V(35)= 30931

W(n) gives the number of five-parts prime numbers in the interval $(10^{n-1}, 10^n)$, where all digits are 3 or 7.
 We think that

$$W(n)=1,381^{n-3}, \text{ where } n \geq 5 \text{ integer.} \tag{2}$$

The V(n) is the factual number of five-parts prime numbers, where all digits are 3 or 7 and the number of five-parts prime numbers calculated according to funktion W (n) (where all digits are 3 or 7) are as follows:

Number of digits	The V(n) factual number in the interval $(10^{n-1}, 10^n)$	The number calculated according to funktion $W(n)=1,381^{n-3}$	V(n)/W(n)
n	V(n)	W(n)	
5	5	1,91	2,62
10	4	9,58	0,42
15	77	48,12	1,60
20	23	241,71	0.10
25	164	1214,12	0,14
30	3898	6098,59	0,64
35	30931	30633,58	1,01

4. Six-parts prime numbers [3], [9], [10], [11],[12].

Definition: a positive integer number is six-parts prime number, if
a/ the positive integer number is prime,
b/ the number of digits is 6k, where k≥1 integer,
c/ dividing it into five equally long parts, every part is a prime.

The six-parts prime numbers are as follows (the last digit can only be 3 or 7):
 {222323, 222337, 222527, 222533, 222553, 222557, 222773, 223253, \
 223273, 223277, 223337, 223577, 223753, 223757, 225223, 225227, \
 225257, 225353, 225373, 225523, 225527, 225733, 227233, 227257, \
 227377, 227533, 227537, 232333, 232357, 232523, 232753, 232777, \
 233323, 233327, 233353, 233357, 233557, 233777, 235273, 235337, \
 235523, 235537, 235553, 235577, 235723, 237233, 237257, 237277, \

237373, 237733, 237737, 252223, 252233, 252253, 252277, 252323, \
 252533, 252727, 252737, 253273, 253537, 253553, 253573, 253733, \
 253777, 255253, 255523, 255733, 255757, 257273, 257353, 272227, \
 272257, 272333, 272353, 272533, 272537, 272737, 272777, 273233, \
 273253, 273323, 273527, 273727, 273773, 275227, 275323, 275357, \
 275573, 275773, 277223, 277273, 277373, 277577, 277757, 322237, \
 322327, 322523, 322537, 322573, 322727, 322757, 323233, 323273, \
 323333, 323377, 323537, 325333, 325537, 325723, 325753, 325777, \
 327277, 327337, 327553, 327557, 327737, 327757, 332273, 332573, \
 333227, 333233, 333253, 333323, 333337, 333533, 333737, 333757, \
 335273, 335323, 335527, 335557, 337223, 337277, 337327, 337537, \
 352237, 352273, 352327, 352333, 352357, 352523, 352753, 352757, \
 353237, 353333, 353527, 353557, 353737, 353777, 355573, 355723, \
 355753, 355777, 357353, 357377, 357727, 357733, 357737, 372223, \
 372277, 372353, 372377, 372523, 372733, 372773, 373273, 373327, \
 373357, 373553, 373753, 373757, 373777,.....etc.

P(n) is the factual number of six-parts prime numbers in the interval $(10^{n-1}, 10^n)$:
 P(6)= 389. Q(n) is the factual number of six-parts prime numbers in the interval $(10^{n-1}, 10^n)$, where all digits are 3 or 7:
 Q(6)= 10 {333337, 333737, 373777, 377737, 733333, 733373, 737773, 773777, 777373, 777737}
 Q(12)= 0 Q(18)= 244 Q(24)= 0 Q(30)= 0 Q(36)= 30790

5. The number of the elements for example of the set of five-parts prime numbers [3], [9],[10], [11], [12].

Let's take the set of Mills' prime numbers!

Definition: The number $m=[M \text{ ad } 3^n]$ is a prime number, where $M=1,306377883863080690468614492602$ is the Mills' constant, and $n=1,2,3,\dots$ is an arbitrary positive integer number. It is already known that the number of the elements of the set of Mills' prime numbers is infinite. The Mills' prime numbers are the following: $m=2, 11, 1361, 2521008887,\dots$

The connection $n \rightarrow m$ is the following: $1 \rightarrow 2, 2 \rightarrow 11, 3 \rightarrow 1361, 4 \rightarrow 2521008887,\dots$ The Mills' prime number $m=[M \text{ ad } 3^n]$ corresponds with the interval $(10^{m-1}, 10^m)$ and vice versa. For instance: $2 \rightarrow (10, 10^2), 11 \rightarrow (10^{10}, 10^{11}), 1361 \rightarrow (10^{1360}, 10^{1361}), \text{ etc. and vice versa. The number of the elements of the set of Mills' prime numbers is infinite. As a consequence, the number of the intervals } (10^{m-1}, 10^m) \text{ that contain at least one Mills' prime number is infinite. The number of five-parts primes in the interval } (10^{m-1}, 10^m) \text{ is } S(m)=1,381^{n-3}.$

The number of five-parts prime numbers is probably infinite: $\lim_{n \rightarrow \infty} V(n) = \infty$ is, if $n \rightarrow \infty$, where $n \geq 5$ integer.

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Conclusion

Countless different sets of special prime numbers have been known. We have found the following set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be task of researchers of the future.

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