International Journal of Latest Engineering Research and Applications (IJLERA) ISSN: 2455-7137

Volume –05, Issue – 11, November 2020, PP – 01-07

Modelling of Rotary Inverted Pendulum based on PID Controller

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Abstract: The aim of this study is to investigate about the rotary inverted pendulum system which is controlled by PID control system. The authors proposed a vertical rotary pendulum rod method based on parameter control. Through analysis of the parameter value of the pendulum rod during vertical and the rod angle as the feedback coefficient the swing time of the pendulum is reduced. This experiment derived the inverted pendulum motion equation of the pendulum rod using the rotation angle regarded as the input.

Keywords: Rotary Inverted Pendulum, PID Control, Pendulum Rod

I. INTRODUCTION

The method to control inverted pendulum was proposed in several common papers [1-3] in control system theory. The interesting main distinctive cases are unstable, nonlinear, non-minimum phase, that affects to robustness, stability margins, and underactuated system. Therefore, the different strategies are used as a benchmark to control the complex system. There are two problems to concentrate on pendulum control, inverted pendulum stabilization and swing-up control design of pendulum.

To achieve the performance and desired system response, we use obtaining dynamic models [4] to control the system problems. The closed-loop control system [5] used to guarantee the robustness and stability. PID control is the most efficient solution and simplest to various control system problems. The steady-state response and transient are controlled by P, I and D functionality. The dynamical system's performance is controlled to be optimal desired. There are several optimal control techniques [6] and optimization [7] presented in the papers for linear [8] and nonlinear dynamical systems [9]. The intelligent computing techniques solve the various problems of the control system such as fuzzy control [10], the sliding mode control [11]. The optimization of intelligent control has been developed by these intelligent computing techniques. These are applied in various control [12] scheme implementation, for example, in inverted pendulum.

This experiment consists of an inverted pendulum installed in a motorized cart. The inverted pendulum experiment is an example commonly used in the control and automation system. Part of its popularity is due to the uncertainties that cannot be controlled. In other words, the pendulum rod will fall if the cart is not moving to balance. The objective of the inverted pendulum control system is to balance pendulum combined by the attached cart force.

II. ROTARY INVERTED PENDULUM MODELING

In this experiment, we consider a two-dimensional problem in which the pendulum is forced to move in the vertical plane indicated in the Fig. 1 and 2.

Fig.1.Rotary inverted pendulum model

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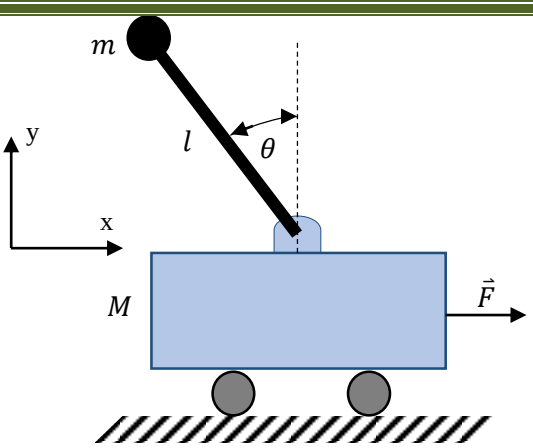

Fig. 2.Rotary inverted pendulum mechanical system

For the inverted pendulum system, the control input is the force F that moves the cart horizontally and the outputs are the angular position of the inverted pendulum θ and the horizontal position of the cart x.

To create appropriate mathematical model and describe the actual object and identification of the parameters, the parameters of mechanical system are defined in Table I.

Table I. Parameter of mechanical system

Parameter	Symbol (Mechanical)
Rod mass	$\boldsymbol{m}$
Cart mass	$\boldsymbol{m_c}$
Distance	$\boldsymbol{l}$
Moment of inertia	$\boldsymbol{I}$
Distance of cart	$\boldsymbol{x}$
Rod angular	$\boldsymbol{\theta}$
Coefficient of the rod	$\boldsymbol{c}$
Coefficient of the cart	$\boldsymbol{b}$
Gravitational acceleration	$\boldsymbol{g}$

The mathematics model for inverted pendulum

$$F - m\ddot{x} - ml\ddot{\theta}\cos\theta + ml\dot{\theta}^2\sin\theta - c\dot{x} = m_c\ddot{x} \tag{1}$$

$$\ddot{x} = \frac{1}{(m_c + m)}\left[-ml\ddot{\theta}\cos\theta + ml\dot{\theta}^2\sin\theta - c\dot{x} + F\right] \tag{2}$$

The links are defined as $\theta \approx 0$, $\cos\theta \approx 1$ and $\boldsymbol{\dot{\theta}^2\theta \approx 0}$, will be calculated

$$\ddot{x} = \frac{1}{(m_c + m)}\left[-ml\ddot{\theta} - c\dot{x} + F\right] \tag{3}$$

$$\ddot{\theta} = \frac{1}{J}\left[m\lg\theta - ml\ddot{x} - b\dot{\theta}\right] \tag{4}$$

Where $J = I + ml^2$

Substitute equation (3), (4) into equation (5) to be solved are

$$\ddot{x} = \frac{1}{(m_c + m)}\left[-ml\left(\frac{1}{J}\left[m\lg\theta - ml\ddot{x} - b\dot{\theta}\right]\right) - c\dot{x} + F\right]$$

$$\ddot{x} = -\frac{m^2l^2g\theta}{J(m_c+m)} + \frac{m^2l^2\ddot{x}}{J(m_c+m)} + \frac{mlb\dot{\theta}}{J(m_c+m)} - \frac{c\dot{x}}{(m_c+m)} + \frac{F}{(m_c+m)}$$

$$\left(1 - \frac{m^2l^2}{J(m_c+m)}\right)\ddot{x} = -\frac{m^2l^2g\theta}{J(m_c+m)} + \frac{mlb\dot{\theta}}{J(m_c+m)} - \frac{c\dot{x}}{(m_c+m)} + \frac{F}{(m_c+m)}$$

$$\ddot{x} = \frac{-m^2l^2g\theta + mlb\dot{\theta} - cJ\dot{x} + JF}{J(m_c+m) - m^2l^2} \tag{5}$$

The rod angular displacement is given by

$$\ddot{\theta} = \frac{1}{J}\left[m\lg\theta - ml\left(\frac{1}{(m_c+m)}\left[-ml\ddot{\theta} - c\dot{x} + F\right]\right) - b\dot{\theta}\right]$$

$$\ddot{\theta} = \frac{m\lg\theta}{J} + \frac{m^2l^2\ddot{\theta}}{J(m_c+m)} + \frac{mlc\dot{x}}{J(m_c+m)} - \frac{mlF}{J(m_c+m)} - \frac{b\dot{\theta}}{J}$$

$$\left(1 - \frac{m^2l^2}{J(m_c+m)}\right)\ddot{\theta} = \frac{m\lg\theta}{J} + \frac{mlc\dot{x}}{J(m_c+m)} - \frac{mlF}{J(m_c+m)} - \frac{b\dot{\theta}}{J}$$

$$\ddot{\theta} = \frac{(m_c+m)m\lg\theta - (m_c+m)b\dot{\theta} + mlc\dot{x} - mlF}{J(m_c+m) - m^2l^2} \tag{6}$$

The state space design is as follows

$$\dot{x} = Ax + BF \tag{7}$$

$$x = \begin{bmatrix} x & \dot{x} & \theta & \dot{\theta} \end{bmatrix}^T \tag{8}$$

$$y = \begin{bmatrix} x & \theta \end{bmatrix}^T \tag{9}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-cJ}{J(m_c+m)-m^2l^2} & \frac{-m^2l^2g}{J(m_c+m)-m^2l^2} & \frac{mlb}{J(m_c+m)-m^2l^2} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{mcl}{J(m_c+m)-m^2l^2} & \frac{(m_c+m)m\lg}{J(m_c+m)-m^2l^2} & \frac{-(m_c+m)b}{J(m_c+m)-m^2l^2} \end{bmatrix} \tag{10}$$

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$$B = \begin{bmatrix} 0 \\ \dfrac{J}{J(m_c + m) - m^2 l^2} \\ 0 \\ \dfrac{-ml}{J(m_c + m) - m^2 l^2} \end{bmatrix} \tag{11}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \tag{12}$$

III. CONTROLLER DESIGN

The PID controller is chosen in this experiment because of its high efficiency and reliability, which consists of three parameters according to proportion, integration and derivatives. The controller calculates the difference between the desired value and the measured value as in the Fig. 3.

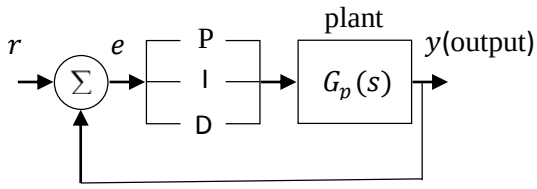

Fig. 3.Flow process with PID controller

The objective of this research is to balance the inverted pendulum automatically by using PID controller. The movement of any object in a circular pattern motion produces continuous angular momentum until it is subjected to external torque. To maintain the stability of the inverted pendulum in an upright position and control the cart in the desired position, the PID control equations are given as

$$u_p = K_{pp}\, e_\theta(t) + K_{ip} \int e_\theta(t)dt + K_{dp} \frac{de_\theta(t)}{dt} \tag{13}$$

Followed by

$$u_c = K_{pc}\, e_x(t) + K_{ic} \int e_x(t)dt + K_{dc} \frac{de_x(t)}{dt} \tag{14}$$

Where, u_p represent the pendulum angle control signal, $e_\theta(t)$ represent the error angle, u_c represent cart control signal, and $e_x(t)$ represent the error position. The changing in any parameters affects the cart position and pendulum angle which occurs tedious tuning after the dynamics are coupled to each other.To tuning the PID controller parameters are completed by observing the responses and using trial & error method from the inverted pendulum model.

IV. RESULT AND DISCUSSION

The PID controller is able to hold the inverted pendulum vertically up in the experiment but it is found not robust enough. The graphical patterns of output values are shown in Fig.4 and Fig.5 respectively. The PID tuning value for experiments are listed in Table II.

Fig. 4.Experiment I, rotary inverted pendulum by PID controller
(a. red color is setpoint, b. blue color is output)

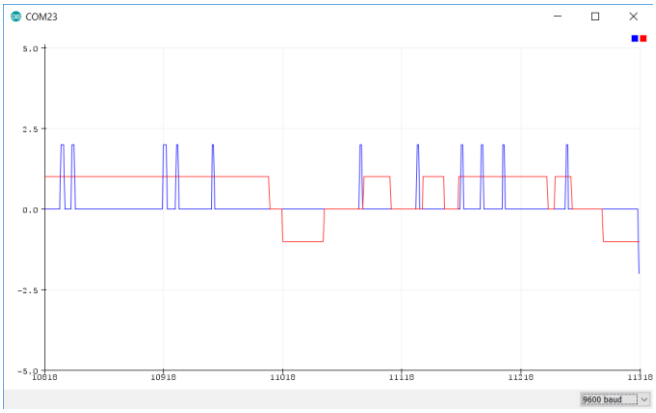

Fig. 5.Experiment II, rotary inverted pendulum by PID controller
(a. red color is setpoint, b. blue color is output)

Table II. PID tuning value

Parameters	Experiment I	Experiment II
Proportional (P)	10	5
Integral (I)	0.8	0.6
Derivative (D)	3.5	1.5

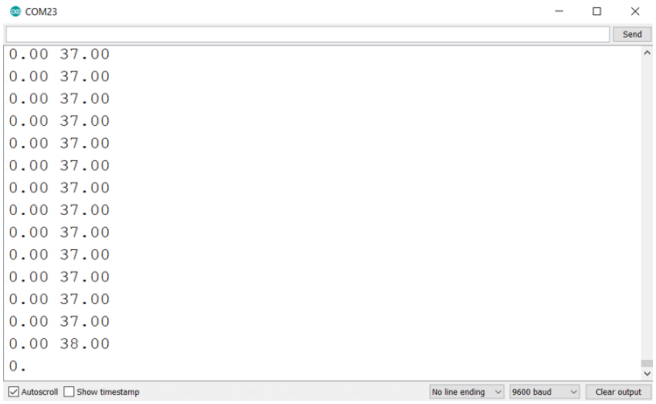

Fig. 6.The value while leaning to the left side

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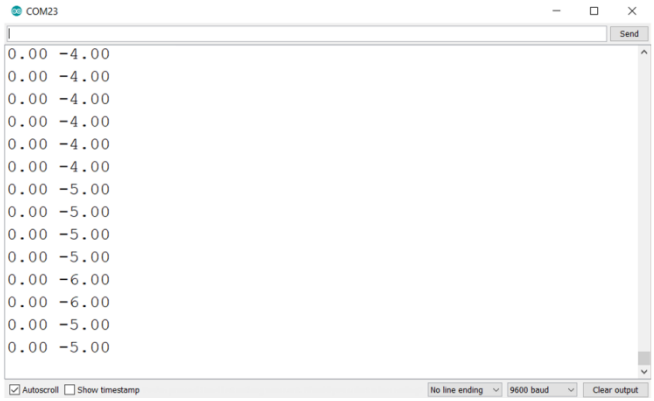

Fig. 7.The value while leaning to the right side

The above Fig. 6 shows the output value of inverted pendulum leaning on the left side, then the system will rotate the rod to the right side to the vertical position. In contrast, if the inverted pendulum leaning to the right side, the controller tries to adjust vertically the pendulum by rotating to the left side as in the Fig. 7.

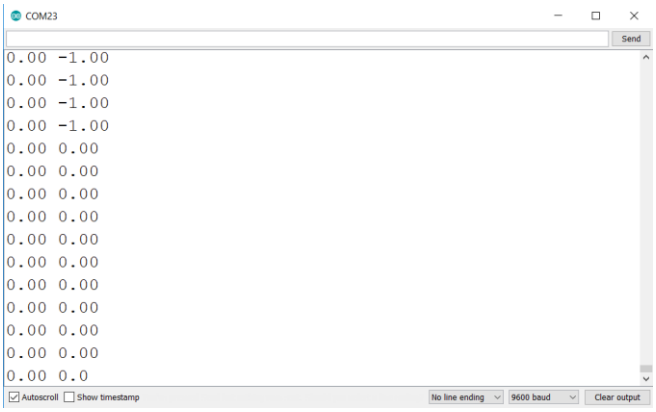

Fig. 8.The value of the inverted pendulum in the stability state

The stability state is the target of the experiment of an inverted pendulum. The value of stability state is shown in the Fig. 8.

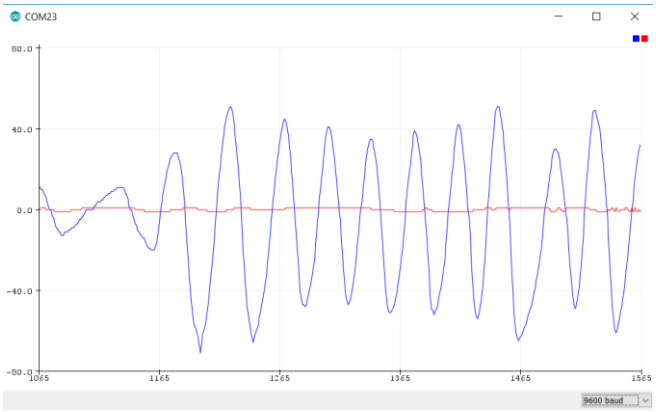

Fig. 9.experimental result (a. red color is setpoint, b. blue color is output)

Fig. 9 shows the experimental result of PID controller that tries to maintain the stability of the inverted pendulum system.

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V. CONCLUSION

In this experiment, the set of PID control method is proposed which is applied to the rotary inverted pendulum system. There are interfaces from the forces outside. It is inconvenient in each time to adjust the parameter's value. The controller for nonlinear objects based on interval methods can be used to synthesize the large group. The aforementioned transformation is carried out using the symmetry control system theory.

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