

Prediction of Compressive Strength of High Strength Concrete Using Multiple Linear Regression Equation (MLRE) and Numerical model

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Abstract: Considerable increase in use of statistics and its methods in representing the complex relationship between factors that contribute to the strength of concrete is noted these days. There is a gap in literature about the statistical and numerical studies of high strength concrete. An effort is made in this research to give an equation that can predict the values of compressive strength of High strength concrete. Moreover, the experimental results and the predicted values of SPSS are validated by using ABAQUS, a finite element modeling tool. The objective of this study is to provide 3D Finite Element model (using ABAQUS) and to use simple data to develop linear regression model (using IBM SPSS) for prediction of Compressive strength of High strength Concrete. HSC was developed by the use of locally available materials such as quartz powder, silica fume, steel fibers and quartz aggregates. The study also investigates the performance of concrete mixtures in terms of Compressive strength at age of 7 and 28 days. The effect of using quartz aggregates in place of quartz powder reduced the compressive strength. Steam curing resulted in reducing the strength when cured above 100 °C due to cross over effect. A data base of 56 concrete cubes (subjected to different proportions of constituents used in HSC) is used as reference to develop equation. The resulting equation was used to find the Compressive strength of HSC and is compared with that of experimental and numerical values obtained. The results showed high correlation coefficient r , 97 % and the coefficient of determination r^2 , 94 % between the actual and predicted values.

Keywords: Compressive Strength; High Strength Concrete (HSC); Regression modeling; Numerical Modeling)

I. INTRODUCTION

High strength concrete is at its acme in construction industry these days. Researchers have defined and improved various parameters (that resulted into development of HPC) in their way [1-5]. Ultimate goal of all these is development of concrete having special performance and that can't always be achieved by using conventional materials, mixing, placing and curing. Oral Buykozturk et.al declared it as concrete having excellent workability, high strength, durability, and high ductility, impervious storage vessels, good hardening and setting time. Dr I.N Patel et al. regarded HPC as Concrete having high strength, high durability, high flow ability, less porosity and less capillarity [6]. Moreover, to validate the experimental findings, a number of improved prediction techniques have been proposed by empirical or computational modeling, statistical techniques and artificial intelligence approaches. Many researchers have concentrated on using multivariable regression models to improve the accuracy of predictions. These multiple linear regression (MLR) models are used to develop relationships for axial capacity of concrete. [7, 8]. Palika Chopra and Dr Maneek Kumar used it for the prediction of compressive strength of concrete with and without fly ash [9]. In combination with artificial neural network multiple regression model has also been used for prediction of strength of mineral concrete [10]. However, the uncertainties of straight lines should be kept in mind and that term must be considered as error term in regression model (Hoffmann, 2010). These uncertainties in using empirical-derived models for design purposes has incited an increased interest in the use of finite element modeling (FEM) to gain a better understanding of the behavior of confined concrete. Many attempts have been made for modeling circular columns subjected to different conditions through the use of computational techniques such as finite element analysis [11-13].

II. EXPERIMENTAL METHOD

About 6 trial mixes were made to achieve the highest strength. The effect of changing the proportion of cement on strength was studied. The percentage weight of cement by weight for all six batches is given in table 1. Mixing regime was kept same with only difference in the quantities is used. The material used in development of High strength Concrete are very expensive so 4"X4"X4" cubes were used for study. It is worth

mentioning that Fine Sand and Silica Sand are kept same in first five batches with quantity slightly reduced in batch 6. Table 2 shows percentage by weight of Fine Sand and Silica Sand. After the casting of cubes for batch 1 it was decided to make some changes in the proportions which were being used previously. As it was observed that after previous casting, approximately it took 48 hours to be hardened. So, it was decided to reduce the amount of super plasticizer. The amount of super plasticizer used in different batches is reported in table 3. The results obtained from 1st and 2nd casting were analyzed. Improvement was noted down in the compressive strength. So it was decided to repeat the experiment with same proportion and improve method of mixing. To improve quality of mixing, plates were attached (welded) to drill machine. There was slight reduction in w/C ratio. To further enhance the results total number of specimens was reduced to 6 in place of 10 this time.

Cement, fine sand, silica sand, silica fume and quartz powder were mixed in tub for 2 minutes as powder. Then water and ½ of super plasticizer was added while mixing the sample in tub for next 2 minutes. The mixture was transferred to concrete mixer and was then allowed to mix for 12 minutes, during the stated time accelerator was added. This was followed by addition of remaining super plasticizer and again mixture was allowed to mix for 5 minutes. Then over 2 minutes all the steel fibers and extra amount of 50 ml super plasticizer was added. The mixture was allowed to mix for 5 minutes until that all the steel fibers were blended in. 9 Cubes were casted because some of the material got lost. After two days cubes were demolded and placed in water for curing. The shown step wise process of mixing is in figure 1 and Figure 2.

III. LINEAR REGRESSION MODEL

Regression models have the advantage that once they get fit, they can predict values fast and accurate than other modeling techniques and are correspondingly simpler to implement in software. When more than one explanatory variable affects the results (output) then multiple regression analysis technique is adapted [14-16].

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon_i \quad (1)$$

Here $\beta_0 + \beta_1 + \beta_2 + \beta_3$ are the co-efficient of linear relationship, $X_1, X_2,$ and $X_3 \dots$ are explanatory variables and ε_i is error term for uncertainties.

III.I. AXIAL CAPACITY PREDICTION EQUATION

Six different trials mixtures were used. The purpose of different proportions of constituents was to study the weightage of each ingredient in strength of concrete. Compressive strength is dependent (outcome) variable and weight of cement; weight of super plasticizer, weight of quartz powder and weight of water are explanatory or independent variables. Equation is developed using IBM SPSS, showing weightage of different ingredients is stated below.

$$P_{n_{HSC}} = 0.183W_c + 0.0254W_{sf} + 0.596W_{QP} - 94.36W_w \quad (2)$$

Here $P_{n_{HSC}}$ stand for Compressive strength of high strength concrete in MPa, W_c represents weight of cement, W_{sf} is actually weight of silica fume, W_{QP} is weight of quartz powder and W_w show weight of water in Kgs for 1 m³ of concrete. The description of model (performance measuring parameters) is shown in Table 5.

IV. NUMERICAL MODELING

ABAQUS Standard 6.12 was used for numerical simulations of concrete cubes. Boundary conditions, different geometric and material parameters were discussed in detail during the modeling of specimens. A homogeneous 3D solid section was used to model concrete material. For the numerical simulations of FRP confined concrete, Drucker-Prager plasticity model was used [14, 15].

ACI code given by Eq. (3) was used for calculation of modulus of elasticity (E_c) of concrete.

$$E_c = 4700\sqrt{f_c'} \quad (3)$$

The density of concrete used was 2.4×10^{-9} tons/mm³. Poisson ratio of 0.2, young moduli of 39500, 43740, 34304, 37305, 40704 and 54000 N/mm² from batch 1 to batch 6 respectively. The initial and maximum increment size of 0.01 and minimum increment size of 10^{-10} is used as input properties.

In the current study, the behavior of concrete material was defined by using the CDP model given by Liu et al. [16] which describes the relationship between inelastic strain, plastic strain, and compression stress and strain of concrete. The damage parameter d_c , plastic strain ϵ_{pl} , and inelastic strain ϵ_{in} are determined using [16]. The CDP model in ABAQUS consists of plastic behavior, compressive behavior and tensile behavior of concrete. Pressure load (UDL) of 25000 is applied on top surface of cube.

The concrete element is modeled using a solid 3-dimensional element in order to display the non-linear behavior of the concrete. The mesh that has been used for the concrete is C3D8R which stand for an 8 node linear brick with reduced integration. The mesh size 10 is being used. Figure 3 shows types of elements, boundary conditions, load and meshing of model.

To define the compressive behavior of concrete in CDP model, it is essential to give inelastic strain ϵ_{in} in further increased degree so that compression failure of concrete can be defined at increased strain and peak elastic stress of concrete is to be provided in the compression behavior. According to Eurocode 2, the compressive stress-strain diagram for concrete has been shown in Figure 4. The linear elastic behavior can be taken up to $0.4f_{cm}$ according to P. Kmiecik et al. [17].

The concrete ultimate tensile strength was estimated by using Eq. (4) proposed by Wang and Vecchio and Genikomsou and Polak [18, 19]. The Modified Tension Stiffening Model is shown figure 5.

$$f_t' = 0.33\sqrt{f_c'} \text{ (Mpa)} \quad (4)$$

V. NUMERICAL MODELING

For Batch 1, the 7th day strength was noted as 55 MPa and it enhanced to 70.5 MPa on 28th Day as given in Table 6. The results were even better for batch 2, with 7th day strength as 76 MPa and 28th Day Strength as 86.6 Mpa reported in table 7. Comparatively low strength was achieved for 3rd Batch as the percentage of cement was increased by about 5% and W/C ratio was decreased to 0.18 as compared to previous two batches. This result in low strength was due to low workability and improper mixing of the ingredients. The strength was noted as 54 Mpa and 66Mpa for 7 days and 28 days respectively as presented in table 8. In batch 4, to study the effect of super plasticizer was main target; this time galnium-51 was used in place of SP-303. The numbers of the specimen were reduced. Unlike the previous batches, in which dry mixing was carried out by driller, this time concrete mixer was used for mixing so that proper blend could be achieved. But once again because of low w/C ratio of 0.2 proper mixing was not achieved that resulted low strengths. The 7th day strength and 28th day strength achieved are 49 Mpa and 63 Mpa as given in table 9. The strength of batch 5 was considerably reduced because sulphate resisting chemicals were added to the water due to personal error. Unlike the previous batches in which the normal curing was done, this time the samples were subjected to steam curing at 90 degree centigrade for more than 48 hours. This showed remarkable changes in the strength. The results are given in table 10. The problems that were faced were in previous batches were controlled in batch 6 that is why 7th day strength achieved was 146.7 MPa the greater of all. However, as these cubes were subjected to steam curing above 100 °C that is why reverse reactions know as cross over effect occurred that resulted in strength reduction for about 10 % on 28th day reported as 132 MPa shown in Table 11. The comparison of axial capacities of batch 1 to batch 6 is shown in figure 6. The values predicted by multiple linear regression equation are close to that of experimental showing an excellent prediction. The percentage difference of predicted values with experimental of Batch 1 to batch 6 are shown in Table 12. The values with negative sign mean values are over estimated. Graphical comparison of predicted values is given in figure 7. The results calculated using ABAQUS are also close to experimental except batch 5 which already mentioned that sulphate addition had negative effect on strength of concrete. The percentage difference for ABAQUS and experimental values are -14% ,14%,-3%, 2%,-16% and 7 % from batch 1 to batch 6 respectively as shown in table 13. The negative sign show that values are over estimated. Table 14 shows comparison of experimental, SPSS and ABAQUS predicted values. The Graphical comparison of experimental, SPSS and ABAQUS values is given in figure 8.

VI. FIGURES AND TABLES

To ensure a high-quality product, diagrams and lettering MUST be either computer-drafted or drawn using India ink.

Figure captions appear below the figure, are flush left, and are in lower case letters. When referring to a figure in the body of the text, the abbreviation "Fig." is used. Figures should be numbered in the order they appear in the text.

Table captions appear centered above the table in upper and lower case letters. When referring to a table in the text, no abbreviation is used and "Table" is capitalized. (10)

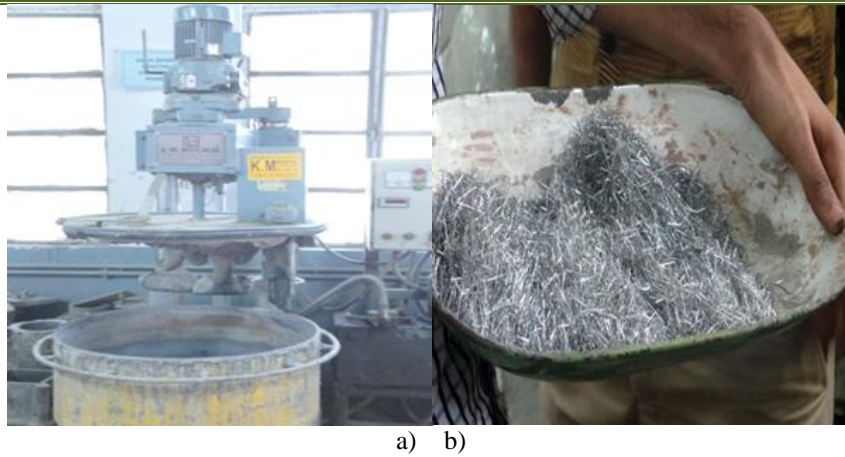


Fig 1.a) Rapid Speed Mixer b) Steel fibres added



Fig 2. c) Blending of fibres in concrete mix d) Concrete poured in

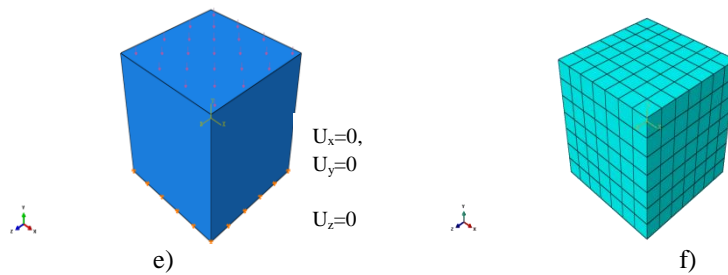


Fig 3.e) Boundary conditions and pressure loading f) Meshing of element

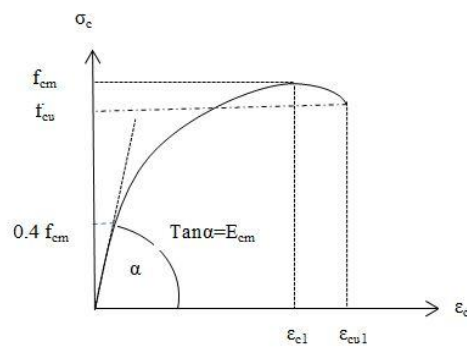


Fig 4.Stress-strain diagram for the analysis of structures, (Eurocode)

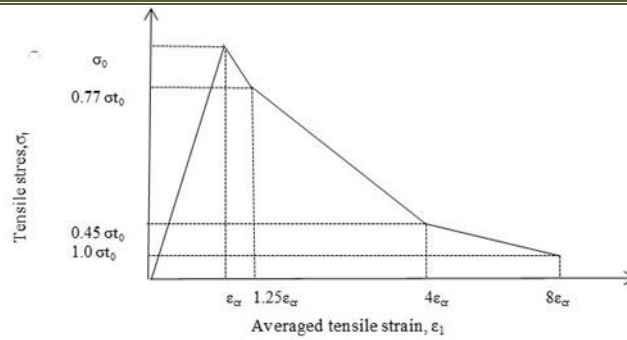


Fig 5. Modified Tension Stiffening Model

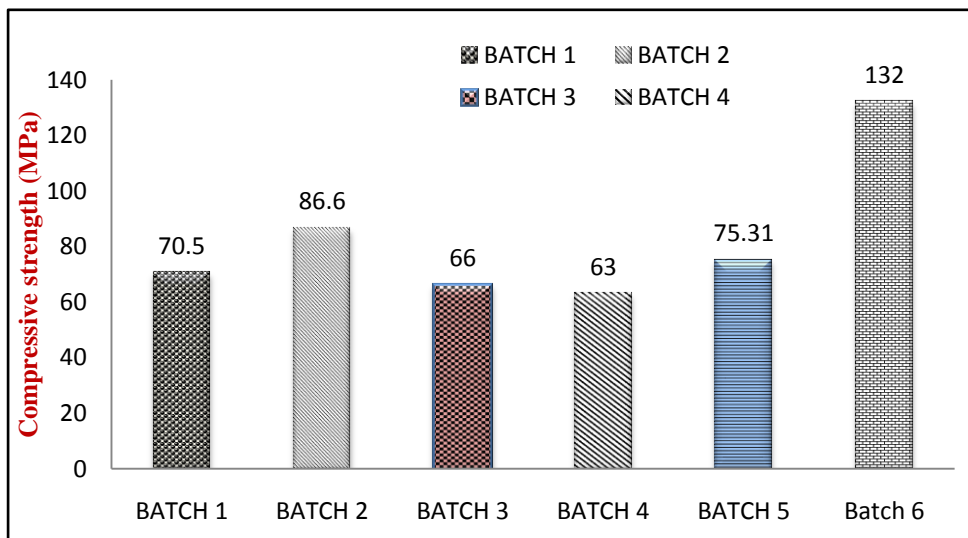


Fig 6: Comparison of experimental values of axial capacity for different batch

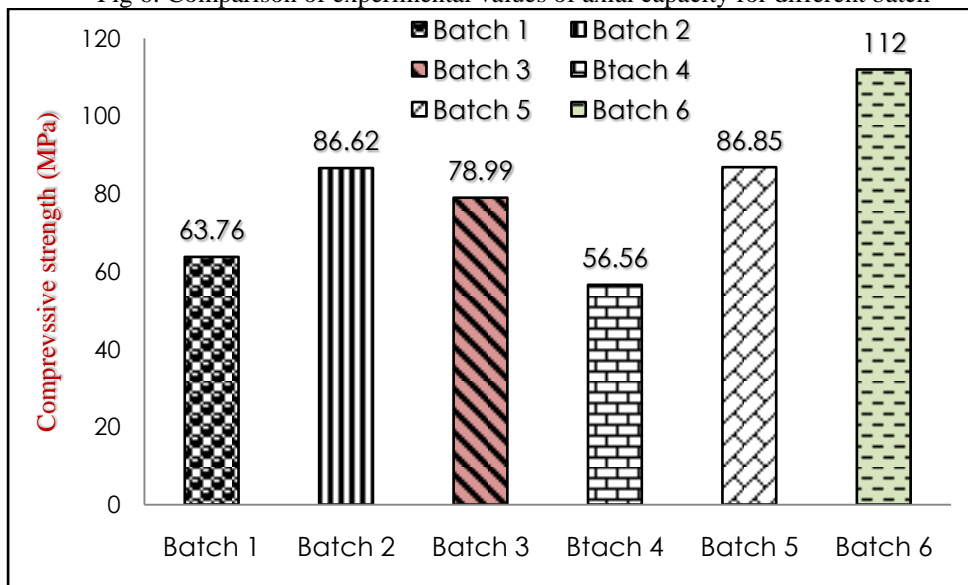


Fig 7: Comparison of SPSS predicted values of axial capacity for different batches

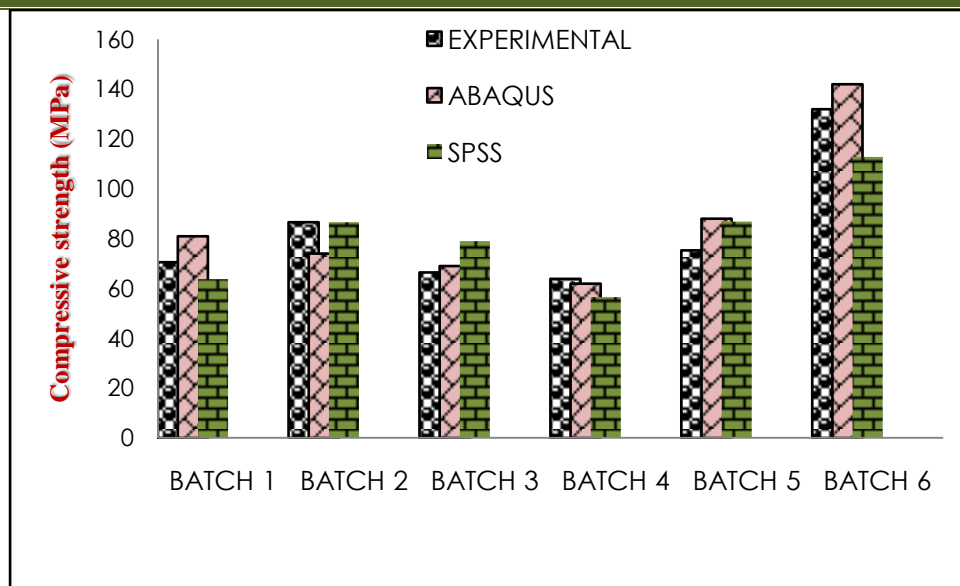


Fig 8: Comparison of Experimental, SPSS and ABAQUS Values

Table 1.Cement content Batch wise

Batch No	Source	Mix design Kg/m ³	Percentage by weight	Estimated Quantity(kg)	Losses (3%)	Total Quantity (Kg)
1	Maple leaf	712	28.5	7.466	.224	7.6901
2	Maple leaf	708	28.3	7.424	0.148	7.5727
3	Maple leaf	754	30.17	7.910	0.119	8.0284
4	Maple leaf	750	30	4.719	0.071	4.7896
5	Maple leaf	442.5	17.7	4.640	.139	4.7793
6	DG Khan Cement	751	30.05	6.334	0.157	6.4913

Table 2.Fine Sand and Silica Sand content Batch wise

Batch No	Material	Source	Mix design Kg/m ³	Percentage by weight	Estimated Quantity(kg)	Losses (3%)	Total Quantity (Kg)
1,2,3,4,5	FineSand 50%	Lawrancepur	1020	40.8	3.209	0.048	3.2569
	Silica sand 50%	Mianwali			3.209	0.048	3.2569
6	Fine Sand (50%)	Lawrancepur	979	39.15	4.126	0.102	4.2285
	Silica Sand (50%)	Mianwali			4.126	0.102	4.2285

Table 3.Silica Fume and Quartz Powder and Super Plasticizer content Batch wise

Batch No	Material	Source	Mix design Kg/m ³	Percentage by weight	Estimated Quantity(kg)	Losses (3%)	Total Quantity (Kg)
1,2,3,4,5,6	Silica Fume	BASF Chemicals	231	9.3	2.422	0.073	2.4950
	Quartz Powder	Imporiant Chemicals	211	8.4	2.213	0.066	2.2790
	Super Plasticizer	BASF Chemicals	30.7	1.2	0.322	0.010	.3316

Table 4.Accelerator, Steel Fibres and Water content Batch wise

Batch No	Material	Source	Mix design Kg/m ³	% by weight	Estimated Quantity (kg)	Losses (3%)	Total Quantity (Kg)
1	Accelerator	BASF Chemical	30	1.2	.322	.010	.3240
	Steel Fibers	Local steel wire	156	6.2	1.636	0.049	1.6849
	Water	Local	109	4.4	1.143	0.068	2.1
2	Super Plasticizer	Imporiant Chemical	30.7	1.2	.322	0.006	0.3284
	Steel Fibers	Local steel wire	156	6.2	1.636	0.033	1.6686
	Water	Local	143	5.72	1.815	0.036	1.8508
3	Steel Fibers	Local steel	50	2	0.524	0.00786	0.5386
	Accelerator	BASF Chemical	30	1.2	0.322	0.005	0.3268
	Water	Local	173	6.92	1.824	0.054	2.02
4	Accelerator	BASF Chemical	30	1.2	0.189	0.003	.1961
	Steel Fibers	Local steel wire	75	3.0	0.472	0.007	.4790
	Water	Local	152.5	6.1	0.959	0.054	2.25
5	Accelerator	BASF Chemical	30	1.2	.315	.009	.3240
	Water	Local	239	9.56	2.506	.061	1.95
	Steel Fibers	Local Steel Wire	100	4	0.843	0.021	0.8641
6	Water	Local	197	7.87	1.659	0.041	1.65

Table 5.Performance measuring Parameters for Regression equation developed.

Performance Measure	Model Data Set
R	0.974
R ²	0.949
RMSE	21.58
Sig.	0.0093

Table 6.28th day compressive strength results Batch-1

Sample #	Load (Ton)	Strength (psi)	Strength (MPa)
1	73	10220	70.5
2	73	10220	70.5
3	71	9940	68.55
4	73	10220	70.5
5	75	10500	72.4
Average			70.5

Table 7. 28th compressive strength results Batch-2

Sample No1	Load (Tons)	Weight	Density (Kg/m ³)	Strength(psi)	Strength (MPa)
1	88	2.382	2271	12320	85
2	91	2.349	2240	12720	88
3	87	2.369	2260	12180	84
4	91	2.372	2262	12740	88
5	91	2.365	2255	12740	88
Average					86.6

Table 8. 28th day compressive strength results Batch-3

Sample #	weight kg	Load Tons	Strength psi	Mpa
1	2.229	88	12320	84.96552
2	2.312	70	9800	67.58621
3	2.302	52	7280	50.2069
4	2.234	54	7560	52.13793
5	2.241	80	11200	77.24138
Average				66.42

Table9. 28th compressive strength results Batch-4

Sample #	weight kg	Load Tons	Strength psi	MPa
1	2.232	64	8960	61.7931
2	2.42912	60	8400	57.93103
3	2.303	74	10360	71.44828
Average				63.81

Table 10. 28th day compressive strength results Batch-5

Sample No1	Load	Weight	Density (Kg/m ³)	Strength(psi)	Strength (MPa)
1	76	2.186	2098	10640	73.37
2	74	2.178	2077	10360	71.44
3	76	2.213	2110	10640	73.37
4	86	2.235	2131	12040	83.03
Average					75.31

Table 11. 28th day compressive strength results Batch-6

Sample #	Weight (kg)	Density (kg/m ³)	Load (tons)	Strength (psi)	Strength (MPa)
1	2.32	2213.74	134	18760	129.3793103
2	2.46	2347.328	124	17360	119.7241379
3	2.19	2089.695	146	20440	140.9655172
4	2.39	2280.534	144	20160	139.0344828
Average					132

Table 7. Predicted values of axial capacity using multiple linear regression equation developed.

Batch	Cement Kg/m ³	Silica fume Kg/m ³	Quartz Powder Kg/m ³	Super plasticizer Kg/m ³	Water Kg/m ³	Predicted Axial Capacity (Mpa)
1	712	231	211	30.7	2.1	63.7634
2	708	231	211	30.7	1.85	86.6214
3	754	231	211	30.7	2.02	78.9982
4	750	231	211	30.7	2.25	56.5634
5	758	275	210	30.7	1.95	86.857
6	751	233	210	30.7	1.65	112.8172

Table 8. Predicted values of axial capacity using ABAQUS

Batch No	Axial Capacity (Mpa)
1	81
2	74
3	91
4	62

Table 9. Comparison of Experimental, SPSS and ABAQUS Values

Bath No	EXP (1)	ABAQUS (2)	SPSS (3)	% diff (1-2)	% diff Exp (1-3)
1	70.5	63.7634	81	9.55	-14.89
2	86.6	86.6214	74	-0.02471	14.549
3	66.42	78.9982	69	-18.93	-3.884
4	63.81	56.5634	62	11.3	2.836
5	75.31	86.857	88	-15.33	-16.85
6	132	112.8172	142	14.53	-7.575

VII. CONCLUSION

1. The maximum compressive strength achieved was 132 Mpa (19140 psi). Such a high strength was achieved due incorporation of high tensile strength steel fibres, silica fume and quartz powder.
2. Steam curing up to 90 °C resulted in increased compressive strength whereas above 100°C the strength reduced due to cross over effect.
3. The result predicted by multiple linear regression equations lies close to that of experimental with percentage difference of below 12 % for all batches. The performance measuring parameters of model are also according to that specified by rules of statics. This confirms that excellent axial capacity prediction equation is developed.
4. The results obtained by using ABAQUS software as a finite element modelling tool shows that best fit model is calibrated. The axial capacity values obtained shows close resembles to that of experimental showing average percentage difference of below 10%.

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