Optimal control law in the missile stage into the geometric dynamic trajectory of the guidance method

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Abstract: The arrangement of elements in the remote missile control station is not suitable between the placement of the launcher and the navigation equipment. The distance between them can be up to hundreds of meters. Missile from launch need to be in the field of view of the radar waves wing. At the time of catching the missile signal, the straight deviation can be very large, due to the direction of the speed vector to the meeting point. At that time, the missile may lose control and pass the radar field of view. Therefore, the paper presents the method of synthesizing optimal control law, to ensure put the missile into dynamic trajectory the fastest. Simple control law, can be applied in practice.

Keywords: Missile, Optimal control law, Geometric dynamic trajectory, Guidance method.

I. INTRODUCTION

The basis of the control problem in the stage of entering the geometric dynamic trajectory is the fast-acting maximum that coincides between the missile's angular position with the center of the radar's field of view and take the missile's speed vector to the right direction conforms to the geometric dynamic trajectory of guidance method [1], [2].

The control stage, at the moment, from the beginning of radio control to the time of starting the motion according to the geometric dynamic trajectory, is called stage into the dynamic trajectory [1], [4].

The main task of the control problem in the stage of entering the dynamic trajectory is to eliminate fastest the deviation between the real trajectory and the required geometric dynamic trajectory [2].

II. OPTIMAL CONTROL LAW

The control problem on the dynamic trajectory entry stage belongs to the class of fast-acting maximal problems.

Straight deviation h, has the form [3], [5]:

$$h = r_m(\varepsilon_m - \varepsilon_t) \tag{1}$$

r_m - Distance of missile;

ε_m - The angle of inclination of the missile;

ε_t - The angle of inclination of the target.

Derivative (1), we have:

$$\dot{h} = \dot{r}_m(\varepsilon_m - \varepsilon_t) + r_m(\dot{\varepsilon}_m - \dot{\varepsilon}_t) = V_m(\varepsilon_m - \varepsilon_t)cos(\theta_m - \varepsilon_m) + V_m sin(\theta_m - \varepsilon_m) - \frac{r_m}{r_t}V_t sin(\theta_t - \varepsilon_t) \tag{2}$$

V_m - Missile velocity;

θ_m - The angle of inclination of the missile's trajectory;

θ_t - The angle of inclination of the target's trajectory;

r_t - Distance of target.

Since $(\varepsilon_m - \varepsilon_t)$ is small, (2) has the form:

$$\dot{h} = V_m\left[\theta_m - \varepsilon_t - \frac{r_m}{r_t}V_t sin(\theta_t - \varepsilon_t)\right] \tag{3}$$

Symbol $w = -\varepsilon_t - \frac{r_m}{r_t}V_t sin(\theta_t - \varepsilon_t)$. Function w determines the motion of the target, or the change of straight deviation, then (3) can be rewritten:

$$\dot{h} = V_m(\theta_m + w) \tag{4}$$

The kinetics of the missile's motion can be described by the equation:

$$\dot{\theta}_m = k.u \tag{5}$$

k - Amplification coefficient (closed loop, missile stabilization);
u - Control command.

Set $x_1 = h, x_2 = \theta_m + w$, equations (4), (5) are rewritten in systematic form:

$$\begin{cases} \dot{x}_1 = V_m.x_2 \\ \dot{x}_2 = k.u + \dot{w} \end{cases} \tag{6}$$

With (6), solve the fast acting maxima problem. According to the dynamic programming method, with the system of differential equations $\dot{x} = f(x,u,t)$, $x(0) = x_0$, control to minimize the functional $I = \int_0^T G(x,u)dt$ satisfying the Belman equation:

$$\min_{u\in\Omega}\left[G(x_0,u_0) + \frac{\partial s}{\partial t} + (gradSf)\right] = 0 \tag{7}$$

S - Minimum value of the function.

For a fast acting optimization problem, the class of functions has the form:

$$I = \int_0^T dt \tag{8}$$

Assumptions, the quantity u in (6) is a standardized quantity for determining domain $|u| \le 1$. Write Belman's equation for (6).

$$1 + \frac{\partial T}{\partial x_1}V_m x_2 + \min_{|u|\le 1}\partial\frac{\partial T}{\partial x_2}(k.u + \dot{w}) = 0 \tag{9}$$

Seeing that, $k > 0$, the optimal control has the form:

$$u^* = -sign\frac{\partial T}{\partial x_2}$$

The equation to find T, has the form:

$$1 + \frac{\partial T}{\partial x_1}x_2 V_m + \frac{\partial T}{\partial x_2}\dot{w} - \left|\frac{\partial T}{\partial x_2}\right|k = 0 \tag{10}$$

Represent region L_1 in phase space, with $u = 1$ and region L_2 with $u = -1$.

The sign change point of the control is determined by the sign of the derivative.

- For region L_1:

$$1 + \frac{\partial T}{\partial x_1}V_m x_2 + \frac{\partial T}{\partial x_2}(k + \dot{w}) = 0$$

$$T = 2\left(\frac{1}{2}x_2^2 - \frac{k+\dot{w}}{V_m}x_1\right)^{1/2} - \frac{x_2}{k+\dot{w}} \tag{11}$$

When $V_m > 0$ and $k + \dot{w} > 0$ (maneuver characteristics of the target smaller than the missile).

Then (11), has the form:

$$\frac{\partial T}{\partial x_1} = \frac{-\dfrac{k+\dot{w}}{V_m}}{\left(\dfrac{1}{2}x_2^2 - \dfrac{k+\dot{w}}{V_m}x_1\right)^{1/2}};\ \frac{\partial T}{\partial x_2} = \frac{x_2}{\left(\dfrac{1}{2}x_2^2 - \dfrac{k+\dot{w}}{V_m}x_1\right)^{1/2}} - \frac{1}{k+\dot{w}}$$

- For region L_2:

$$1 + \frac{\partial x}{\partial x_1}V_m x_2 + \frac{\partial T}{\partial x_2}(-k + \dot{w}) = 0$$

$$T = 2\left(\frac{1}{2}x_2^2 + \frac{k - \dot{w}}{V_m}x_1\right)^{1/2} + \frac{x_2}{k - \dot{w}};\ \frac{\partial T}{\partial x_2} = \frac{x_2}{\left(\frac{1}{2}x_2^2 + \frac{k - \dot{w}}{V_m}x_1\right)^{1/2}} + \frac{1}{k - \dot{w}} \tag{12}$$

Replace (12) into (11), get:

$$1 - \frac{(k + \dot{w})x_2}{\left(\frac{1}{2}x_2^2 - \frac{k + \dot{w}}{V_m}\right)^{1/2}} + \dot{w}\left(\frac{x_2}{\left(\frac{1}{2}x_2^2 - \frac{k + \dot{w}}{V_m}x_1\right)^{1/2}} - \frac{1}{k + \dot{w}}\right) - \left|\frac{\partial T}{\partial x_2}\right| = 0 \tag{13}$$

To satisfy equation (12), then:

$$\frac{k}{k + \dot{w}} - \frac{k.x_2}{\left(\frac{1}{2}x_2^2 - \frac{k + \dot{w}}{V_m}\right)^{1/2}} \geq 0$$

Or

$$\left(\frac{1}{2}x_2^2 - \frac{k + \dot{w}}{V_m}x_1\right)^{1/2} \geq \left(k + \dot{w}\right)x_2$$

When $k + \dot{w} > 0$ and hypothesis $x_2 > 0$, we get:

$$\left[\left(k + \dot{w}\right)^2 - \frac{1}{2}\right]x_2^2 + \frac{k + \dot{w}}{V_m}x_1 \leq 0$$

With $(k + \dot{w})^2$ □ $\frac{1}{2}$, get the control switching line equation.

$$x_1 \leq -V_m\left(k + \dot{w}\right)x_2^2 \tag{14}$$

If $x_2 < 0$, then: $\frac{1}{2}x_2^2 - \frac{k + \dot{w}}{V_m}x_1 \geq 0$

Therefore, the switching line equation will be determined by:

$$x_1 \leq \frac{V_m}{2\left(k + \dot{w}\right)}x_2^2 \tag{15}$$

Similarly, for region L_2, get:

$$x_2 > 0;\ x_1 \leq \frac{-V_m}{2\left(k + \dot{w}\right)}x_2^2 \tag{16}$$

$$x_2 < 0;\ x_1 \leq V_m\left(k - \dot{w}\right)x_2^2 \tag{17}$$

When $x_2 > 0$, choose the smaller sign between (14) and (16). When $x_2 < 0$ choose the larger sign (15) and (17). Get the switch line:

$$x_1 = -V_m\left(k + \dot{w}\,sign(x_2)\right)x_2^2\,sign(x_2) \tag{18}$$

The optimal control law has the form:

$$u = -sign\left[x_1 + V_m\left(k + \dot{w}\,sign(x_2)\right)x_2^2\,sign(x_2)\right] \tag{19}$$

Replace $x_1 = h, x_2 = \dot{h}$ (suitable for (6)) into (19), get:

$$u = -sign\left[h + \frac{k + \dot{w}\,sign(\dot{h})}{V_m}\dot{h}^2\,sign(\dot{h})\right] \tag{20}$$

To eliminate unstable motion around the origin of the phase plane, the Parabola on the phase plane is approximated by two straight lines.

$$F(h) = \begin{cases} h & When\quad |h| < h_{max} \\ h_{max}\,sign(h) & When\quad |h| \geq h_{max} \end{cases} \tag{21}$$

Then, the control law is unique during the control time.

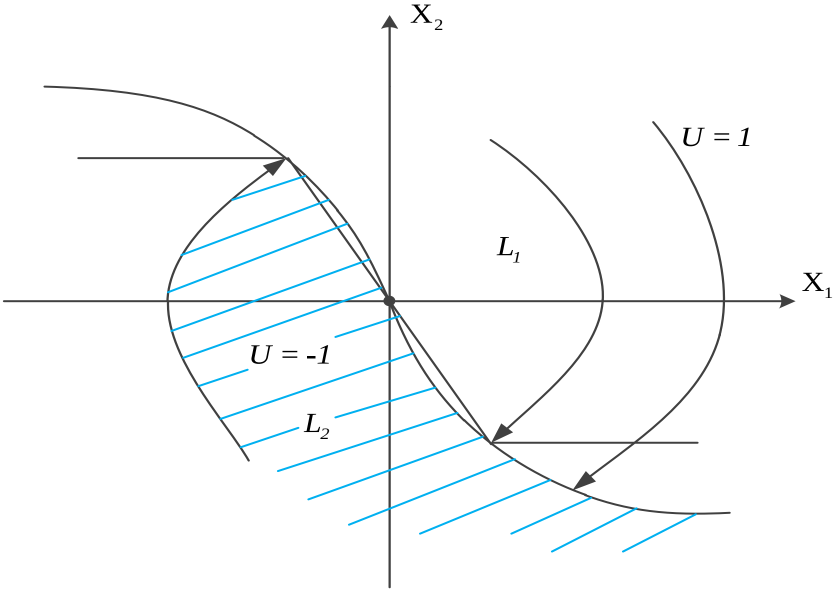

Figure 1. Phase image for optimal control

III. SIMULATION RESULTS AND ANALYSIS

The algorithm survey is performed within the remote missile control loop [2], [5], [5]. The simulation organization diagram is in the form of figure 2, with the parameters are selected follows:

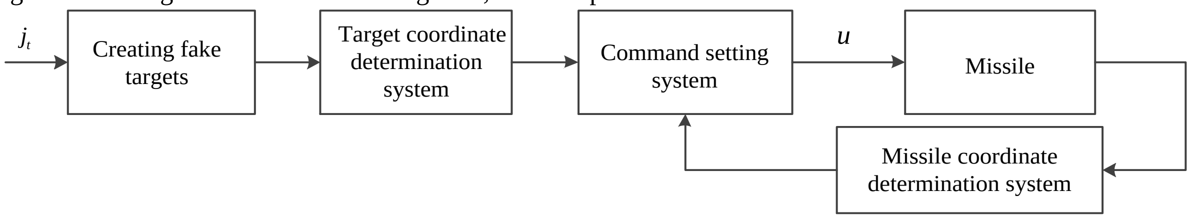

Figure 2. Simulation organization diagrams

- Target has speed $V_t = 450(m/s)$, flying in, the tilt distance $D = 20(km)$, altitude $H = 5,1(km)$, maneuver start time at moment $t_1 = 4(s)$, maneuver finish at moment $t_2 = 6(s)$, maneuver $30(m/s^2)$.

- Missile velocity $V_m = 720(m/s)$.

- The command setting system creates command according to the 3-point guidance method.

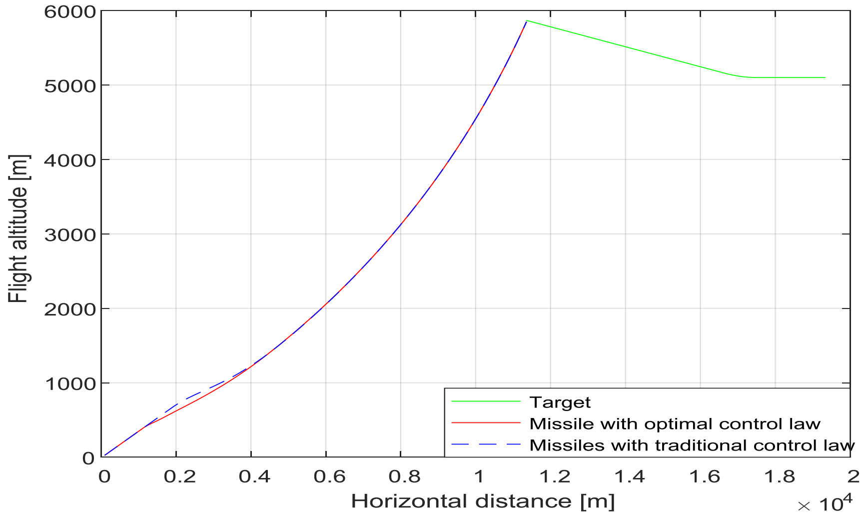

Figure 3. Trajectory of missile - target

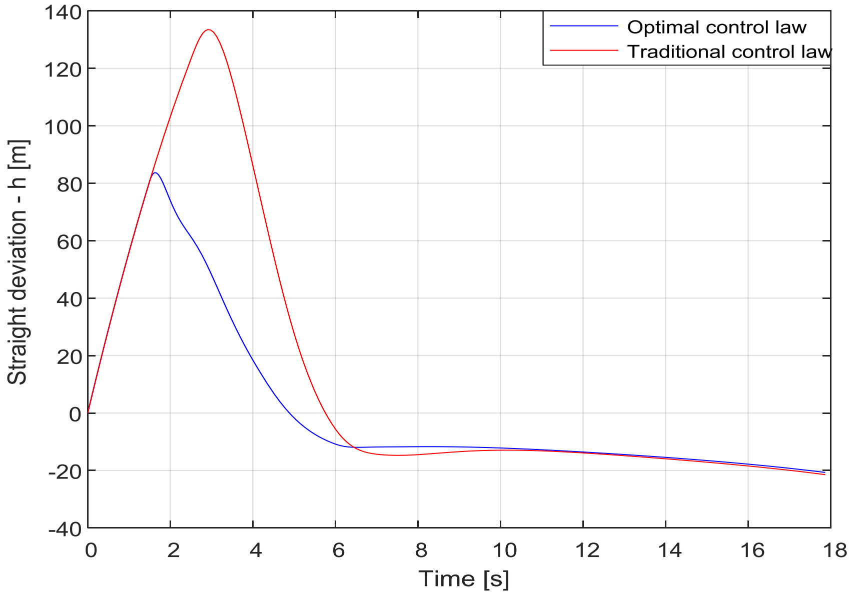

Figure 4. Straight deviation

From the simulation results, we see: With optimal control law, the missile into the fastest dynamic trajectory, the straight deviation at the meeting point is small. With the traditional control law, the straight deviation is larger, in some cases, the control may be lost.

IV. CONCLUSIONS

This article presents a method of synthesizing the optimal control law in the stage of putting the missile into dynamic trajectory. Simple control law, reliable simulation results, can be applied in practice. With the optimal control law, ensure the missile always into the fastest dynamic trajectory, the error at the meetting point is the smallest. Ensure the missile is always controlled during the dynamic trajectory entry stage.

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