Predicting the Flow of Methane in Coal Mine Using Fick's and Darcy's Law Models

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Abstract: This work considered the development and simulation of models that can predict the flow of CH_4 gas in coal bed using Fick's and Darcy's laws. The models were developed from first principles and solved using central difference approximation (CDA) method, since the developed models were partial differential equations in nature. The following operating parameter were obtained from five different coal mines in United States of America Department of Interior: A (Marshall County Mine), B (Enlow Fork Mine), C (River View mine), D (Tunnel Ridge Mine) and E (Ohio County Mine). The operating data obtained were: temperature, pressure, porosity, diffusivity, permeability, and density of coal matrix. Fick's law at a diffusivity of 0.11556m^2/h, 0.0756m^2/h, 0.1746m^2/h, 0.25508m^2/h, 0.32364m^2/h, and porosity of 0.0271, 0.0235, 0.0305, 0.0383, and 0.0399 gave methane gas recovery of 80.3%, 77.98%, 81.99%, 82.39% and 82.96% respectively. Darcy's law at permeability of 1.6876 x $10^{-13}m^2$, 2.4772 x $10^{-13}m^2$, 2.8127 x $10^{-13}m^2$, 3.1217 x $10^{-14}m^2$, 3.4315 x $10^{-14}m^2$, and porosity of 0.0235, 0.0271, 0.0305, 0.0383 and 0.0399 gave methane gas recovery rate of 68.57%, 72.92%, 75.95%, 77.17% and 79.44% respectively. Fick's and Darcy's law were compared with plant data from literature for five different coal mines. A to E. It was observed that Fick's law was dominant for mines B, and D while Darcy's law was dominant for mines, A,C and E in terms of having the lowest percent deviation from plant data for methane recovery percent. This is because the more fracture the coal bed is more dominant Darcy's law applies, since mass transport through fracture is governed by permeability constant which is a constant in Darcy's law, also the more the porosity of the coal mine the more dominant Fick's law applies because mass transport through coal (micropore structure) is governed by diffusivity which is a constant in Fick's law.

Keywords: Fick's law, Darcy's law, methane, simulation, coal bed

1. Introduction

Pam & Connel (2012), stated that Coal mine emissions make up approximately 3% of the world's anthropogenic methane emissions, and the quantity of methane emissions from coal mining alone is over 25 million tons every year. Roughly 70% of the methane emissions come from coal mine ventilation air methane (VAM), which is not only a greenhouse gas but also a wasted energy resource if not utilized. Ruyin (2014), stated that the existence of coal bed gas in colliery could be adsorbed gas, free gas, and dissolved gas respectively. Adsorption occurs when the methane molecules adhere to the internal surface of coal micropores. Cleat matrices are usually filled with water and have a porosity in the range of 0.1–1%. Even though the value is small, it directly contributes to production, (Zhang, et al., 2016). Mixed genetic gas means a mixture of gas of different genesis types, showing different geochemical characteristics from single genesis type gas. Methane shows biogenic genesis, ethane is a sign of thermal genesis, N 2 mainly comes from atmosphere and has the characteristics of mixed genesis gas (Song-Yan, 2012). There are different classification for CBM genesis type based on experimental analysis of carbon isotopes, hydrogen isotopes and R o index. Secondary biogenic gas, thermal degradation gas, thermal cracking gas and mixed genetic gas and the relevant tracing indicator system is established, (Ayers, 2002). Coal bed methane (CBM) is a type of gas present in active working mine sites, this gas is extracted from the air in the coal mine helping improve safety and preventing uncontrolled release of methane to atmosphere (Mou et al., 2015).

Dagde & Ehirim (2017), developed a model capable of predicting the single-phase flow of methane through coal as a porous media. The model was developed by applying the principles of conservation of mass on a controlled volume of coal seam and incorporating the Darcy's law for lamina flow of methane. The model was solved numerically using implicit formulation of finite difference method. Predictions were made on the sensitivity of the model by varying parameters such as permeability, cleat distance, temperature, porosity, viscosity and the partial pressure. The result obtained showed that an increase in permeability led to an increase in effective stress and decrease in the flowrate.

Adel *et al.*(2016), summarized coal bed methane (CBM) as a term giving to methane gas produced or emitted in association with coal mining activities either from the coal seam itself or from other gassy formation underground. The amount of coal bed methane (CBM) generated at a specific operations depends on the productivity of the coal mine. Coal bed methane can be capture by engineer that augment the mine ventilation system or it can be emitted into the mine environment and exhausted from the mine shafts along with ventilation air.

Bingxiang & Weyong (2017), stated that coal mine methane is a combination of methane and air released during the process of coal mining and must be vented for safety reason. The reason methane is used in gas engine it is because of it significant benefits of the environments as it greenhouse effect is twenty one time higher than that of carbon dioxide. Coal is obtained through mining which in term release methane which are trapped within the coal seam into the atmosphere. Methane emission can also occur from the collapse of the nearby rock after part of coal seam has been mined and the artificial roof and wall that hold it is removed as mining progress to another section, and the debris released from the collapse methane into the atmosphere which in turned become hazardous to the environment.

Thakur *et al.,* (1996), summarized coal bed methane (CBM) as primary clean energy source of natural gas and green energy supply for addressing the universal energy problem. However the recovery of economically viable amounts of methane require some technically production enhance techniques, and they suggested that gas transport behavior in a coal seam is the governing factor for enhanced coal bed methane (ECBM) recovery which includes absorption/adsorption and diffusion in the matrix.

2. Materials and Methods

2.1 Materials

The materials used in this study were obtained from five different coal mine in the U.S Department of the interior in addition to MATLAB R2020a software, journals and Chemical Engineers Handbook.

2.2 Method

The following methods shall be adopted in this research.

2.2.1 Development of Mathematical Model of Darcy's Law from First Principle

Before proceeding with the development of the model, certain assumptions have to be made as follows;

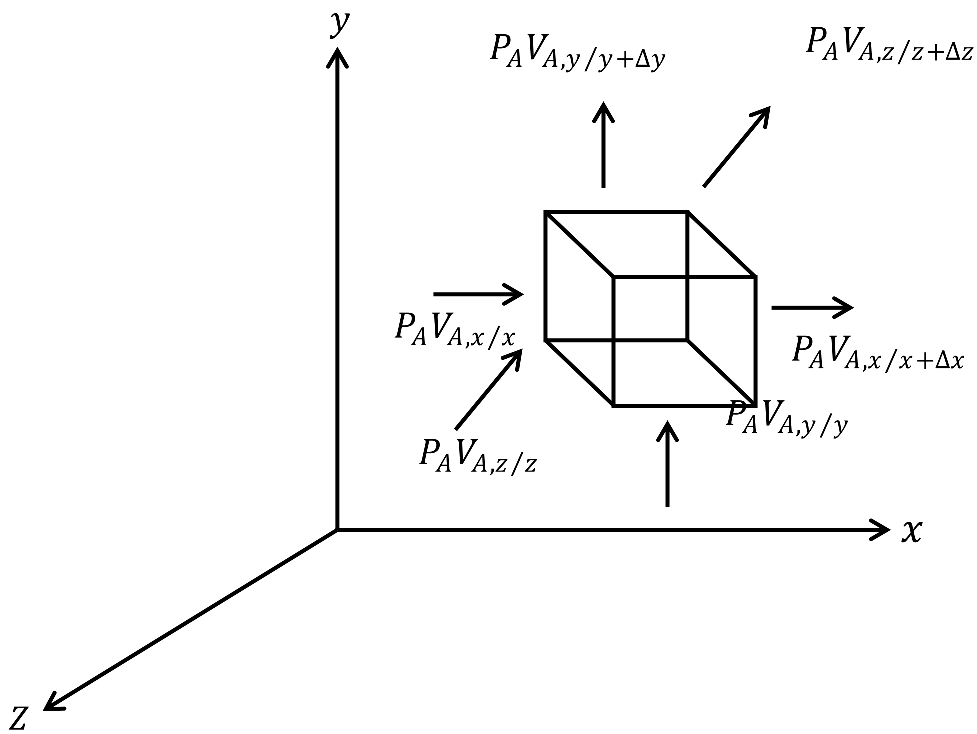

Figure 1: Hypothetical representation of coal body with flow of methane

i. Both Fick's and Darcy's laws are applicable inside the cleat.
ii. The width and cleat spacing remains constant during the gas flow period.
iii. No accumulation of methane at the fracture surface.
iv. During desorption of methane there exist no matrix shrink
v. Permeability is assumed to be constant while it varies as per Klingenberg law
vi. The coal body is a rectangular prison
vii. Gravitational effects are negligible
viii. Isothermal conditions is assumed during degasification process.

Under these conditions, the general relation for mass balance of methane flow in and out of the control volume for a rectangular prison without chemical reaction may be stated as:
The general material balance for Darcy's law is written as follows:

$$\begin{bmatrix} Netrateofmassflux \\ ofMethanewithinthe \\ controlvolume \end{bmatrix} + \begin{bmatrix} Netrateofaccumulation \\ ofMthanewithinthe \\ controlvolume \end{bmatrix} \pm \begin{bmatrix} Rateofchemical \\ production/depletion \\ ofmethanewithinthe \\ controlvolume \end{bmatrix} = 0 \quad (1)$$

The various terms in equation 1 are defined as follows:

$$\frac{\partial \rho_A V_{AX}}{\partial x} + \frac{\partial \rho_A V_{AY}}{\partial y} + \frac{\partial \rho_A V_{AZ}}{\partial z} + \frac{\partial \rho_A \emptyset}{\partial t} = 0 \quad (2)$$

Assuming that flow of methane is in the z-direction only, we have

$$\frac{\partial \rho_A V_{AZ}}{\partial z} + \frac{\partial \rho_A \emptyset}{\partial t} = 0 \qquad (3)$$

From Darcy's law

$$V_{AZ} = -\frac{KA\partial P_A}{\mu \partial z} \qquad (4)$$

Recall that one mole of methane contains a mass equivalent to its molecular weight; thus

$$\rho_A = C_A M_A \qquad (5)$$

$$C_A = \frac{P_A}{RT} \qquad (6)$$

Therefore,

$$\rho_A = \frac{P_A M_A}{RT} \qquad (7)$$

Substituting (4) and (7) into equation (3) gives

$$\frac{\partial}{\partial z}\left[-\rho_A \frac{KA\partial P_A}{\mu \partial z}\right] + \frac{\partial}{\partial t}\left[\frac{P_A M_A}{RT}\right]\emptyset = 0 \qquad (8)$$

$$-\rho_A \frac{KA}{\mu}\frac{\partial^2 P_A}{\partial^2 Z^2} + \emptyset \frac{M_A}{RT}\frac{\partial P_A}{\partial t} = 0$$

Let $\rho_A \frac{KA}{\mu} = \alpha \; and \; \emptyset \frac{M_A}{RT} = \beta$

$$-\alpha \frac{\partial^2 P_A}{\partial^2 Z^2} + \beta \frac{\partial P_A}{\partial t} = 0 \qquad (9)$$

$$-\frac{\partial^2 P_A}{\partial^2 Z^2} + \frac{\beta}{\alpha}\frac{\partial P_A}{\partial t} = 0 \qquad (10)$$

2.2.2 Development of Mathematical Model of Fick's law from First Principle

The general relation for mass balance of methane flow in and out of control volume for a cube is stated as follows:

$$\begin{bmatrix} Netrateofmolarflux \\ ofMethanewithinthe \\ controlvolume \end{bmatrix} + \begin{bmatrix} Netrateofaccumulation \\ ofMthanewithinthe \\ controlvolume \end{bmatrix} \pm \begin{bmatrix} Rateofchemical \\ production/depletion \\ ofmethanewithinthe \\ controlvolume \end{bmatrix} = 0 \qquad (11)$$

$$\frac{\partial J_A}{\partial x} + \frac{\partial J_A}{\partial y} + \frac{\partial J_A}{\partial z} + \frac{\partial C_A \emptyset}{\partial t} \qquad (12)$$

Assuming that methane flows in the z-direction

$$\frac{\partial J_A}{\partial z} + \frac{\partial C_A \emptyset}{\partial t} \qquad (13)$$

The flow of methane in a porous media obey Fick's law which states that the molar flux of a fluid through a porous media is proportional to the concentration gradient.

$$J_{AZ} = -\frac{D\partial C_A}{\partial z} \quad (14)$$

Substituting equation (14) into (13) gives

$$\frac{\partial}{\partial z}\left[-\frac{D\partial C_A}{\partial z}\right] + \frac{\partial C_A \emptyset}{\partial t} = 0 \quad (15)$$

$$-D\frac{\partial^2 C_A}{\partial z^2} + \frac{\emptyset \partial C_A}{\partial t} = 0 \quad (16)$$

$$\frac{\partial^2 C_A}{\partial z^2} = \frac{D}{\emptyset}\frac{\partial C_A}{\partial t} \quad (17)$$

Let $\omega = \frac{D}{\emptyset}$

$$\frac{\partial C_A}{\partial t} = \omega \frac{\partial^2 C_A}{\partial z^2} \quad (18)$$

2.2.3 Solution Technique

Central difference approximations solution technique was used to simulate the developed models for both second and first order differential.

2.2.4.1 Solution Technique for Darcy's Law

Figure 1 shows the flow chart/algorithm used for solving the developed model

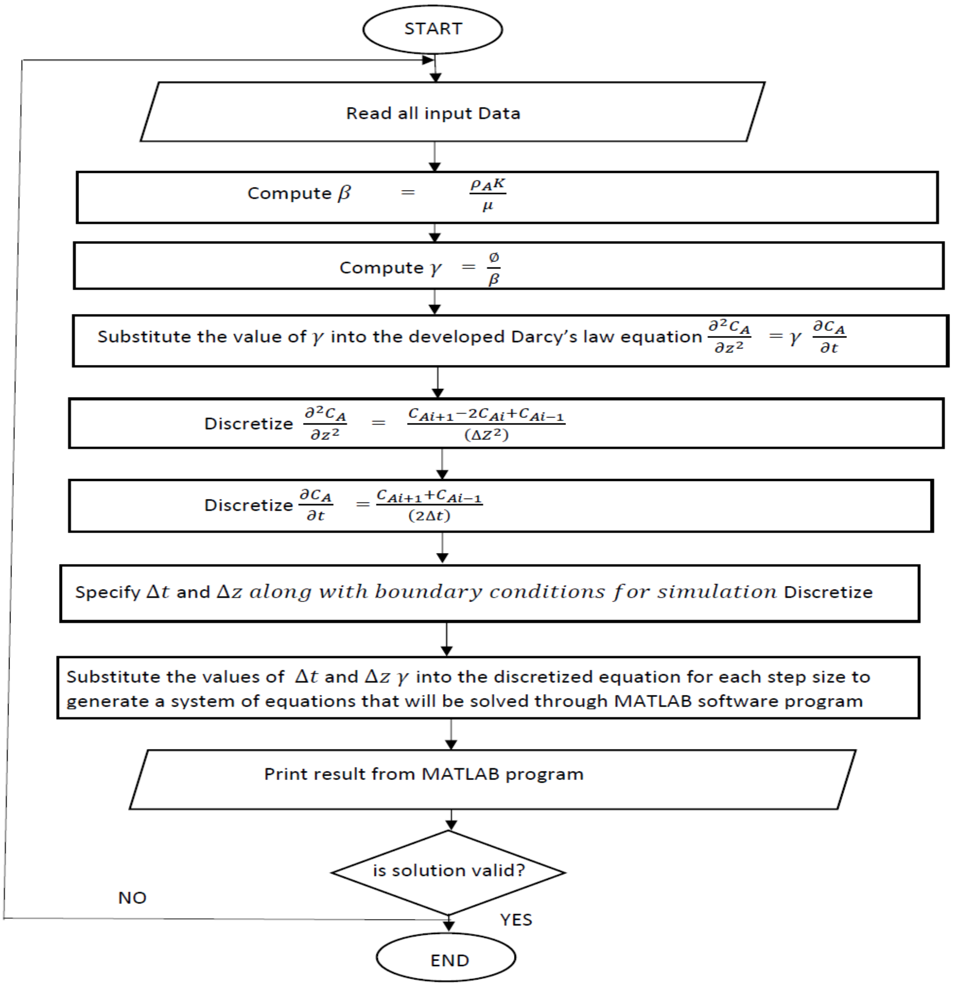

Figure 1: Darcy's law Solution Flow Chart

2.2.5.1 Solution Technique for Fick's Law

Figure 2 shows the flow chart/algorithm used for solving the developed model

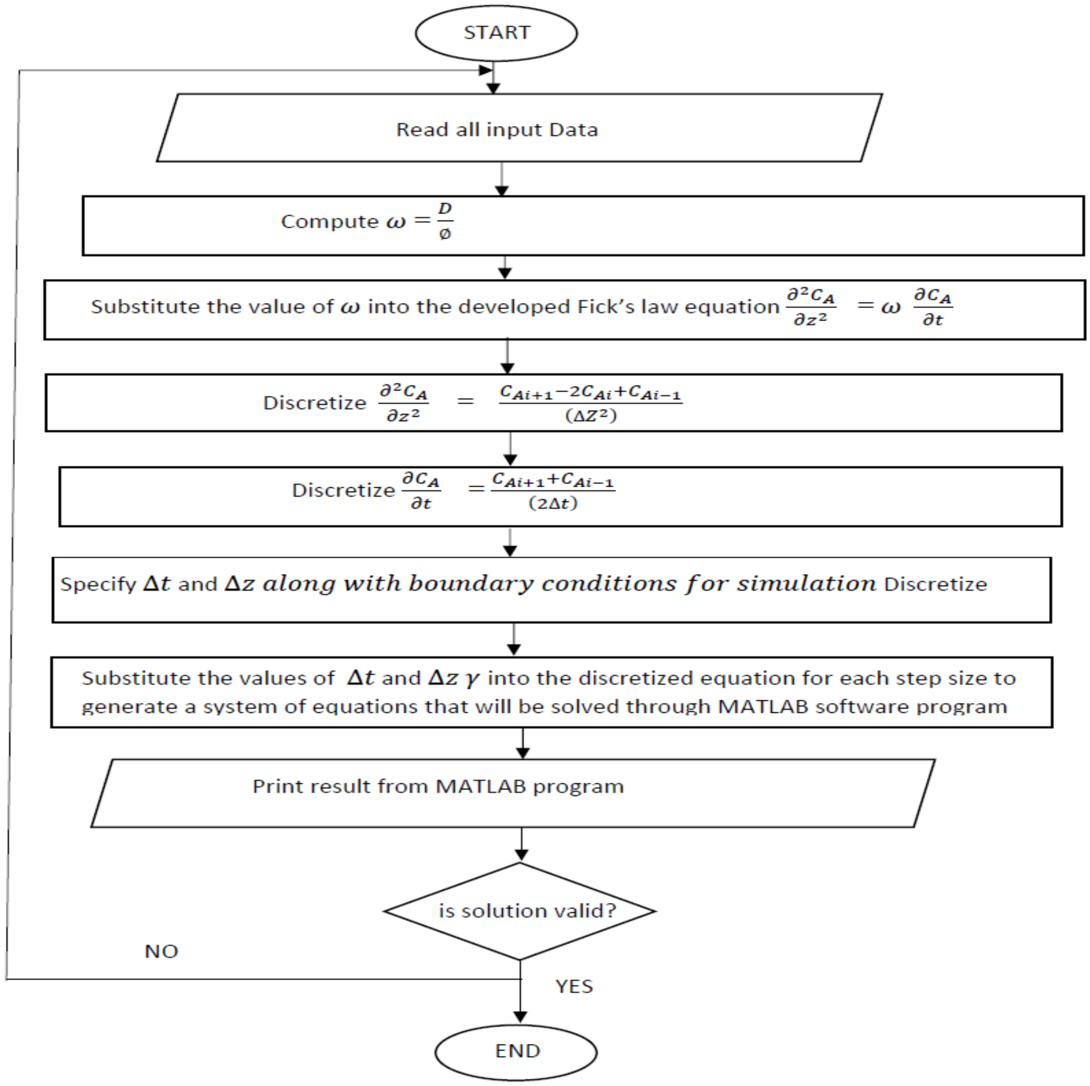

Figure 2: Fick's Law Solution Flow Chart

2.2.5 Boundary Conditions

The boundary conditions used for the simulation of the coal mine is as follows : the depth of the mine at datum level is Z_o = 0m and the final depth of the mine is Z_f = 600m, while the step size for the depth is ΔZ = 100m, the initial time at datum level when mining is just about to take place is t_o = 0 minutes and the final time at the final depth of the mine is t_f = 250 minutes while the time step size is Δt = 40 minutes. The initial composition (mole fraction in percent) of methane at the datum level when mining was just commencing is C_{ai} = 0% while the maximum composition (mole fraction in percent) is C_{Af}= 100% while the composition step size is 10%.

3. Results and Discussion

The result obtained from the simulation of Darcy and Fick's law model which were developed from the principles of conservation of mass otherwise known as material balance. The developed model equations were partial differential in nature and as such central difference approximation method through the application of MATLAB software program are presented below:

Table 1: Comparison of Fick's and Darcy's Law in Terms of Methane Content Recovery

Temperature (K)	Gas Diffusivity (m2/h)	Porosity	Permeability (m2)	Coal Mine Type	Methane Content (%) for Fick's Law	Methane Content (%) for Darcy's Law
293	0.0756	0.0235	1.6876×10^{-13}	A	77.98388	68.57274
299	0.11556	0.0271	2.4772×10^{-13}	B	80.33858	72.92852
301	0.1746	0.0305	2.8127×10^{-13}	C	81.98971	75.94793
303	0.24408	0.0383	3.1217×10^{-14}	D	82.38678	77.17204
305	0.32364	0.0399	3.4315×10^{-14}	E	82.9643	79.44266

Table 1 shows the comparison of Fick's and Darcy's law model in terms of methane content recovery for five different coal mine and at different process parameter such as: diffusivity, porosity and Permeability. Fick's law model seems to have the highest recovery rate of methane content for the five coal mines than Darcy's law model, however this can only be verified when both results are compared with actual plant data to know which model is predominant in each coal mine by calculating the percent deviation from the actual data.

Table 2: Comparison of Fick's and Darcy's Law Model with Plant Data for Methane Content Recovery

Methane Content (%) for Fick's Law	Methane Content (%) for Darcy's Law	Actual Plant Data (%)	Coal Mine Type	Percent Deviation (%) for Fick's	Percent Deviation (%) for Darcy's
77.98388	68.57274	70	A	11.40	2.04
80.33858	72.92852	78	B	2.99	6.50
81.98971	75.94793	73	C	12.31	4.04
82.38678	77.17204	80	D	2.98	3.53
82.9643	79.44266	81	E	2.43	1.92

Table 2: shows the comparison of methane content recovery for coal mines A to E, for Fick's and Darcy's model prediction respectively. Darcy's model had the least percent deviation for coal mine A and hence we can conclude that Darcy's model is predominant in the case scenario in terms of predicting methane content recovery more accurately than Fick's model. Fick's model had the least percent deviation for coal mine B and is more predominant in this case scenario, Darcy's model had the least percent deviation for coal mine C and is more predominant in this case scenario, Fick's model had the least percent deviation for coal mine D and is more predominant in this case scenario, and finally Darcy's model had the least percent deviation for coal mine E and is more predominant in this case scenario. The reason for this is because the more fracture the coal bed the more dominant will be Darcy's law since mass transport through fracture is governed by permeability constant which is a constant in Darcy's law while the more the porosity of the coal mine the more dominant will be Fick's law because mass transport through coal (micropore structure) is governed by diffusivity which is a constant in Fick's law. (Mou*et al*., 2015).

3.2 Comparison of Fick's and Darcy's Laws

The results obtained from the model that was developed for both Darcy's and Fick's laws and solved in MATLAB using central difference approximations are presented in figures 4.21 to 4.22

3.2.1 Comparison of Darcy's and Fick's Laws for Coal Mine A

Fick's and Darcy's law comparison for coal mine A is presented in figure 3. The essence of the comparison is to see the maximum methane content (%) that each law can predict at a depth of 500m. To ascertain which law is predominant in coal mine A then the result obtain for both laws must be validated with the actual plant data for coal mine A.

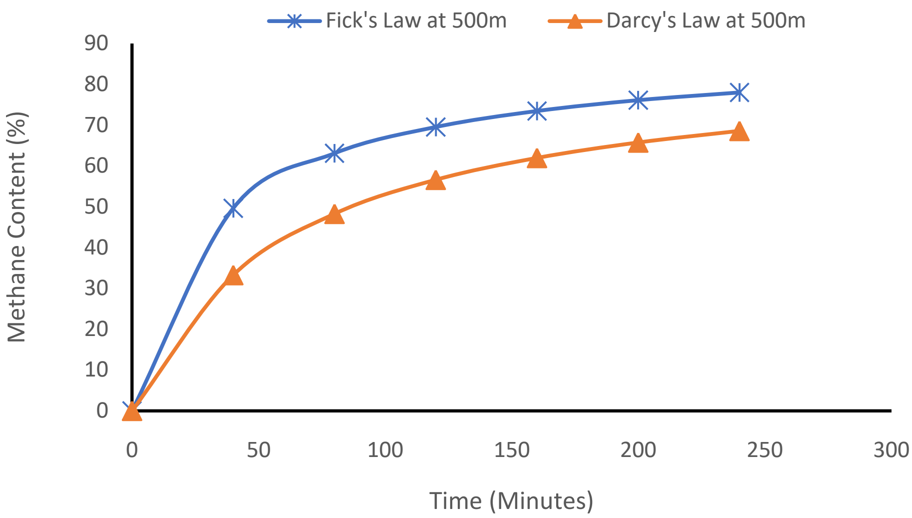

Figure 3: Comparison of Fick's and Darcy's for coal mine A

In figure 3 the results obtained for both Fick's and Darcy's were plotted. Fick's law predicted the maximum methane content of 77.98% at a depth of 500m while Darcy's law predicted a maximum methane content of 68.57%. While it may appear that Fick's law predicted a higher methane conte than Darcy's law, to be fully certain which law is predominant in coal mine A, then both laws model prediction were compared with the actual plant data obtained for coal mine A in terms of percent deviation and it was observed that Darcy's law gave the least percent deviation of 2.04%, hence we concluded that the law that is predominant in coal mine A is Darcy's law which would have been very difficult to predict unless with the help of the model prediction for both laws developed in this work.

3.2.2 Comparison of Darcy's and Fick's Laws for Coal Mine B

Fick's and Darcy's law comparison for coal mine A is presented in figure 4 The essence of the comparison is to see the maximum methane content (%) that each law can predict at a depth of 500m. To ascertain which law is predominant in coal mine B then the result obtain for both laws must be validated with the actual plant data for coal mine B.

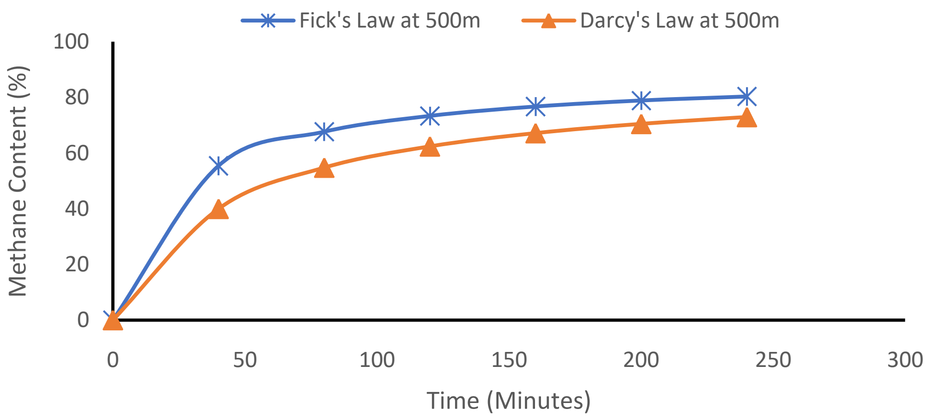

Figure 4: Comparison of Fick's and Darcy's for coal mine B

In figure 4 the results obtained for both Fick's and Darcy's were plotted. Fick's law predicted the maximum methane content of 80.33% at a depth of 500m while Darcy's law predicted a maximum methane content of 72.92%. Although Fick's law predicted a higher methane content than Darcy's law, to be fully certain which law is predominant in coal mine B, then both laws model prediction were compared with the actual plant data obtained for coal mine B in terms of percent deviation and it was observed that Fick's law gave the least percent deviation of 6.50%, hence we concluded that the law that is predominant in coal mine B is Fick's law which would have been very difficult to predict unless with the help of the model prediction for both laws developed in this work.

3.2.3 Comparison of Darcy's and Fick's Laws for Coal Mine C

Fick's and Darcy's law comparison for coal mine A is presented in figure 5. The essence of the comparison is to see the maximum methane content (%) that each law can predict at a depth of 500m. To ascertain which law is predominant in coal mine C then the result obtain for both laws must be validated with the actual plant data for coal mine C.

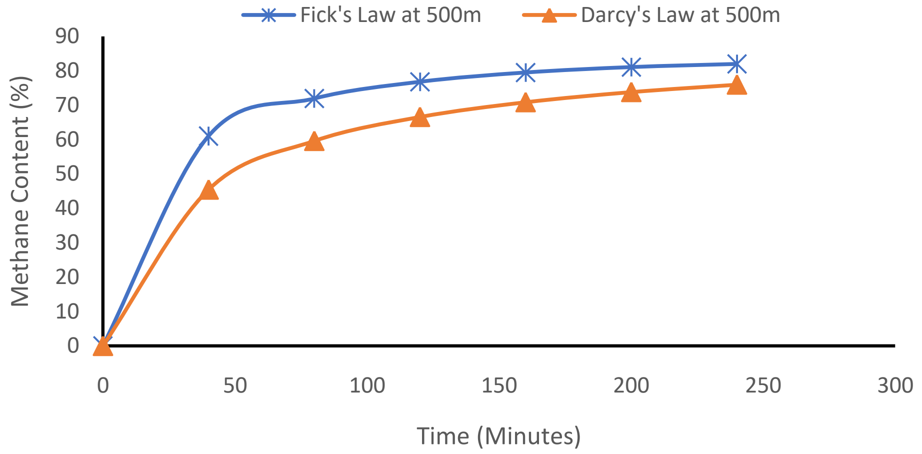

Figure 5: Comparison of Fick's and Darcy's for coal mine C

In figure 5 the results obtained for both Fick's and Darcy's were plotted. Fick's law predicted the maximum methane content of 81.99% at a depth of 500m while Darcy's law predicted a maximum methane content of 75.94%. While it may appear that Fick's law predicted a higher methane conte than Darcy's law, to be fully certain which law is predominant in coal mine C, then both laws model prediction were compared with the actual plant data obtained for coal mine C in terms of percent deviation and it was observed that Darcy's law gave the least percent deviation of 4.04%, hence we concluded that the law that is predominant in coal mine A is Darcy's law which would have been very difficult to predict unless with the help of the model prediction for both laws developed in this work.

3.2.4 Comparison of Darcy's and Fick's Laws for Coal Mine D

Fick's and Darcy's law comparison for coal mine A is presented in figure 6. The essence of the comparison is to see the maximum methane content (%) that each law can predict at a depth of 500m. To ascertain which law is predominant in coal mine D then the result obtain for both laws must be validated with the actual plant data for coal mine D.

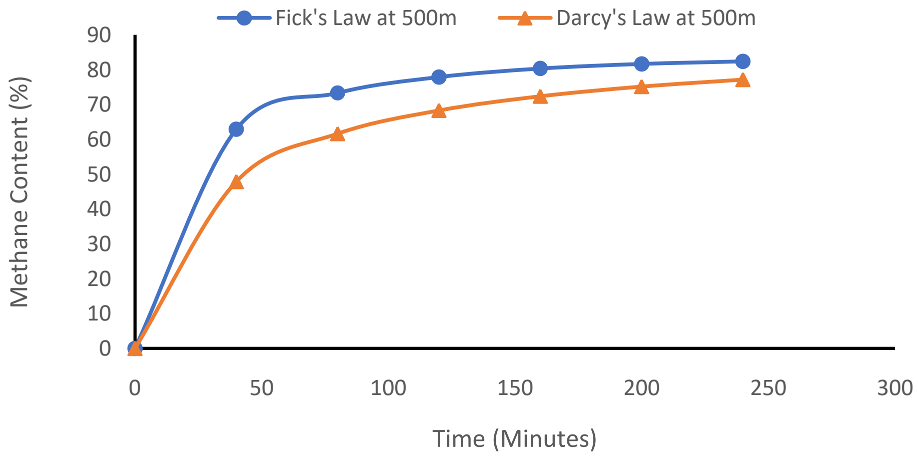

Figure 6: Comparison of Fick's and Darcy's for coal mine D

In figure 6 the results obtained for both Fick's and Darcy's were plotted. Fick's law predicted the maximum methane content of 82.39% at a depth of 500m while Darcy's law predicted a maximum methane content of 77.17%. While it may appear that Fick's law predicted a higher methane content than Darcy's law, to be fully certain which law is predominant in coal mine D, then both laws model prediction were compared with the actual plant data obtained for coal mine D in terms of percent deviation and it was observed that Fick's law gave the least percent deviation of 2.98%, hence we concluded that the law that is predominant in coal mine D is Fick's law which would have been very difficult to predict unless with the help of the model prediction for both laws developed in this work.

3.2.5 Comparison of Darcy's and Fick's Laws for Coal Mine E

Fick's and Darcy's law comparison for coal mine A is presented in figure 7. The essence of the comparison is to see the maximum methane content (%) that each law can predict at a depth of 500m. To ascertain which law is predominant in coal mine E then the result obtain for both laws must be validated with the actual plant data for coal mine E.

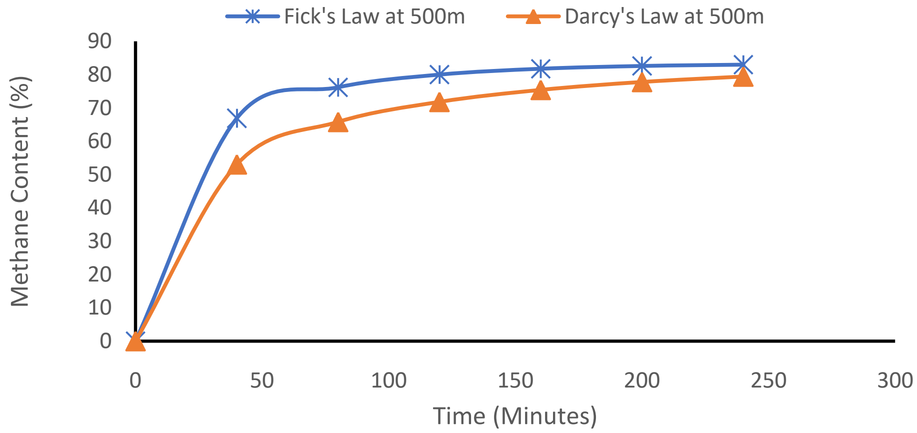

Figure 7: Comparison of Fick's and Darcy's for coal mine E

In figure 7 the results obtained for both Fick's and Darcy's were plotted. Fick's law predicted the maximum methane content of 82.96% at a depth of 500m while Darcy's law predicted a maximum methane content of 79.44%. While it may appear that Fick's law predicted a higher methane conte than Darcy's law, to be fully certain which law is predominant in coal mine E, then both laws model prediction were compared with the actual plant data obtained for coal mine E in terms of percent deviation and it was observed that Darcy's law gave the least percent deviation of 1.92%, hence we concluded that the law that is predominant in coal mine E is Darcy's law which would have been very difficult to predict unless with the help of the model prediction for both laws developed in this work.

4. Conclusion

Fick's and Darcy's mass transport models capable of predicting methane content (%) recovery with depth and time was developed from the principles of conservation of mass and solved using central difference approximation method through the aid of MATLAB software application. Five coal mines coded A to E were used to validate the two mass transport models to ascertain which model was more accurate and efficient for a given coal mine. From literature coal beds or mines are classified according to the mode of mass transport law that is dominant in the coal bed, hence three categories exist: those in which Fick's law is dominant, those in which Darcy's law is dominant and those having a combination of both mass transport laws at equal rate but this is rare in nature but the first two cases can easily occur. No coal mine exhibit mass transport by either Fick's or Darcy's law alone but both are always present since methane has affinity for coal and therefore permeability and diffusivity must occur simultaneously since Fick's law uses Diffusivity, Darcy's law uses permeability.

Out of the five coal mines considered, Fick's law transport model was found to be dominant in coal mines B and D respectively while Darcy's law was dominant in coal mines A, C and E respectively. The reason for this is because the more fracture the coal bed the more dominant will be Darcy's law since mass transport through fracture is govern by permeability constant which is a constant in Darcy's equation while the porosity of the coal mine the more dominant will be Fick's law because mass transport through coal (micropore structure) is governed by diffusivity which is a constant in Fick's law.

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