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Some Results on Belief Functions induced by Probability Mass Function

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Abstract: Instead of representing a parameter with a singular numerical value, it is advantageous to express it as an interval, with the endpoints serving as lower and upper bounds. The belief function and plausibility function act as lower and upper bounds, respectively, for the probability function. Consequently, investigating the properties of belief and plausibility functions becomes crucial. In this paper, we employ a novel basic belief assignment derived from the probability mass function of a discrete probability distribution. Our focus is on examining various properties of both the belief and plausibility functions.

Keywords: probability mass function, basic probability assignment, Belief function, Plausibility function, probability, Bayesian belief function.

I. INTRODUCTION

In the realm of uncertainty, every occurrence in our daily lives adheres to some known or unknown probability distribution. Therefore, the selection of an appropriate probability distribution plays a crucial role in decision-making. Understanding the common characteristics of all probability distributions becomes imperative.

This paper introduces a novel transformation that converts a probability mass function into a basic belief assignment. We explore this new transformation along with other functions related to belief functions. Additionally, we present a summary of the properties of this transformation that can identify its true nature.

The structure of the paper unfolds as follows: initially, in section 2 we provide an overview of the fundamentals of discrete belief functions and probability functions. Section 3 delves into the properties of the new transformation, while Section 4 deduces properties concerning belief and plausibility functions. Finally, we conclude our research paper and include a list of references. In this manner, we comprehensively cover the preliminaries of discrete belief functions and probability functions.

II. PRELIMINARIES

Frame of Discernment: Dictionary meaning of Frame of Discernment is frame of good judgment insight. The word discern means recognize or find out or hear with difficulty. From Shafer‘s book [6], if frame of discernment $\mathbf{\Theta}$ is

$$\mathbf{\Theta} = \{\theta_1,\ \theta_2, \cdots,\ \theta_n\}$$

then every element of $\mathbf{\Theta}$ is a proposition. The propositions of interest are in one -to -one correspondence with the subsets of$\mathbf{\Theta}$. The set of all propositions of interest corresponds to the set of all subsets of$\mathbf{\Theta}$, denoted by $2^{\mathbf{\Theta}}$.

If $\mathbf{\Theta}$ is *frame of discernment*, then a function $m: 2^{\mathbf{\Theta}} \to [0, 1]$ is called **basic probability assignment** whenever $m(\phi) = 0$ and $\sum_{A\subset\mathbf{\Theta}} m(A) = 1$. The quantity $m(A)$ is called $A's$**basic probability number** and it is a measure of the belief committed exactly to A.The *total belief* committed to A is sum of $m(B)$, for all proper subsets B of A. A function $Bel: 2^{\mathbf{\Theta}} \to [0, 1]$ is called **belief function** over $\mathbf{\Theta}$if it satisfies $Bel(A) = \sum_{B\subset A} m(B)$[2, 3]. If $\mathbf{\Theta}$is a frame of discernment, then a function $Bel: 2^{\mathbf{\Theta}} \to [0, 1]$is *belief function* if and only if it satisfies following conditions

1. $Bel(\phi) = 0$.
2. $Bel(\mathbf{\Theta}) = 1$.
3. For every positive integer n and every collection $A_1, A_2, \cdots, A_n$ of subsets of $\mathbf{\Theta}$

$$Bel(A_1 \cup A_2 \cup \cdots \cup A_n) \geq \sum_{I\subset\{1,2,\cdots,n\}} (-1)^{|I|+1} Bel(\bigcap_{i\in I} A_i) \tag{1}$$

A subset of a frame $\mathbf{\Theta}$is called a **focal element** of a belief function Bel over $\mathbf{\Theta}$if $m(A) > 0$. The union of all the focal elements of a belief function is called its **core**.The quantity $Q(A) = \sum_{A\subset B, B\subset \mathbf{\Theta}} m(B)$ is called **commonality number** for A which measures the total probability mass that can move freely to every point of A. A function $Q: 2^{\mathbf{\Theta}} \to [0, 1]$ is called **commonality function** for Bel. Also $Bel(A) = \sum_{B\subset \bar{A}} Q(A)$ and $Q(A) = \sum_{B\subset A} (-1)^{|B|} Bel(\bar{B})$ for all $A \subset \mathbf{\Theta}$.

Degree of doubt:

$$Dou(A) = Bel(\overline{A}) \; or \; Bel(A) = 1 - Dou(\overline{A}) \; and \; Pl(A) = 1 - Dou(A) = \sum_{A\cap B\neq \phi} m(B) \tag{2}$$

which expresses the extent to which one finds A credible or plausible [6]. We have relation between belief function, probability mass (or density) function and plausibility function is$Bel(A) \leq p(A) \leq Pl(A), \forall A \subset \mathbf{\Theta}$ [2, 3, 7]. A function $P : \Theta \to [0,1]$ [1] is called probability function if

1. $\forall A \subset \mathbf{\Theta}, 0 \leq p(A) \leq 1$.
2. $p(\mathbf{\Theta}) = 1$.

A set function μ[1] on a frame of discernment $\mathbf{\Theta}$is a measure if, it satisfies the following three conditions:

1. $\mu(A) \in [0, \infty], \forall A \in \mathbf{\Theta}$.
2. $\mu(\phi = 0)$.
3. Additive property:
   Forall collections $A_1, A_2, \cdots, A_n, \cdots$

$$\mu\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{I\subset 1,2,\cdots,n,\cdots, I\neq\phi} (-1)^{|A|+1} \mu(\bigcap_{i=1}^{\infty} A_i) \tag{3}$$

The measure is finite or infinite if, $\mu(\mathbf{\Theta}) < \infty$or $\mu(\mathbf{\Theta}) = \infty$. It is probability measure if $\mu(\mathbf{\Theta}) = 1$ [1]. From Shafer‘s book [6], we have Bayesian belief function defined as: If $\mathbf{\Theta}$is frame of discernment then a function $Bel: 2^{\mathbf{\Theta}} \to [0, 1]$ is called Bayesian belief function if

1. $Bel(\phi) = 0$,
2. $Bel(\mathbf{\Theta}) = 1$,
3. $Bel(A \cup B) = Bel(A) + Bel(B)$ whenever $A, B \in \mathbf{\Theta}$ and $A \cap B = \phi$.

Suppose $Bel: 2^{\mathbf{\Theta}} \to [0, 1]$ is belief function. Then following statements are equivalent:

1. Bel is Bayesian.
2. All of $Bel's$ focal elements are singletons.
3. Bel awards a zero commonality number to any subset containing more than one element.
4. $Bel(A) = 1 - Bel(\overline{A})$ for all $A \subset \mathbf{\Theta}$.

III. A NEW BASIC BELIEF ASSIGNMENT INDUCED BY PROBABILITY MASS FUNCTION

In this section, we apply a novel basic belief assignment derived from the probability mass function of a discrete probability distribution, along with findings [5] that are substantiated by theorems and results outlined in the work of Hall and Knight[4].

When $\mathbf{\Theta} > 1$ then we observe that sum of probabilities of all subsets is not equal to 1. But if we find a generalized formula about repeation of singleton set of $\mathbf{\Theta}$in all subsets of $\mathbf{\Theta}$. We have following result as:

Theorem 3.1 If $|\mathbf{\Theta}| = n$ then every element in frame of discernment $\mathbf{\Theta}$is repeated exactly 2^{n-1} number of times and sum of probabilities of all subsets of $\mathbf{\Theta}$is 2^{n-1}.

Remark 1 If $|\mathbf{\Theta}| = n$ then by counting principle $\sum_{\theta_i\in\mathbf{\Theta}} p(\theta_i) = \sum_{i=1}^{n} p(\theta_i)$.

Now consider,

$$\sum_{A\subseteq\mathbf{\Theta}} p(A) = \sum_{A\subseteq\mathbf{\Theta}} \sum_{\theta_i\in A} p(\theta_i)$$

Then,

$$2^{n-1} = \sum_{A\subseteq\mathbf{\Theta}} \sum_{\theta_i\in A} p(\theta_i)$$

That is,

$$\sum_{A\subseteq\mathbf{\Theta}} \sum_{\theta_i\in A} p(\theta_i) = 2^{n-1} \tag{4}$$

Remark 2 Also, we observe that if $\mathbf{\Theta} = n$ then any $\{\theta_i\} \in \mathbf{\Theta}$ is repeated 2^{n-1} times i.e. $\{\theta_i\}$appears 2^{n-1} times in subsets of $\mathbf{\Theta}$. Therefore the probability corresponding to $\{\theta_i\} \in \mathbf{\Theta}$ is added 2^{n-1}times. Also, if $|\mathbf{\Theta}| = n$ then $\sum_{A\in\mathbf{\Theta}} p(A) = 2^{n-1}$. Hence in order to get $\sum_{A\in\mathbf{\Theta}} p(A) = 1$, we have to divide each probability entry by a quantity 2^{n-1}.

Therefore

$$m(\{\theta_i\}) = \frac{p(\{\theta_i\})}{2^{n-1}}, \forall\, \theta_i \in \mathbf{\Theta} \tag{5}$$

Remark 3 Now, let $A = \{\{\theta_1\}, \{\theta_2\}, \cdots, \{\theta_n\}\} \subseteq \mathbf{\Theta}$. In discrete space, since singletons are disjoint hence the intersection of any number of singleton subsets of $\mathbf{\Theta}$is always empty set. Therefore

$$\begin{aligned} p(A) &= p(\{\theta_1\} \cup \{\theta_2\} \cup \cdots \cup \{\theta_n\}) \\ &= p(\{\theta_1\}) + p(\{\theta_2\}) + \cdots + p(\{\theta_n\}) \\ &= 2^{n-1} m(A) \end{aligned}$$

where,

$$m(A) = \sum_{\{\theta\}\in A} m(\{\theta\})$$

Therefore, we get

$$m(A) = \frac{p(A)}{2^{n-1}}, \forall A \subseteq \mathbf{\Theta} \tag{6}$$

Now, we check some properties which are satisfied by this new transformation:

Theorem 3.2 The function $m: 2^{\mathbf{\Theta}} \to [0,1]$ defined by (6), $m(A) = \frac{p(A)}{2^{n-1}}$ is a basic probability assignment.

By using above bba (6), we have $m(A) = \frac{p(A)}{2^{n-1}}$, the belief function $Bel: 2^{\mathbf{\Theta}} \to [0,1]$ becomes

$$\begin{aligned} Bel(A) &= \sum_{B\subset A} m(B) \\ &= \sum_{B\subset A} \frac{p(B)}{2^{n-1}} \end{aligned} \tag{7}$$

Here, we give some deductions of some theorems from Shafer‘s book [6] by using (6).

Theorem 3.3 The belief function $Bel: 2^{\mathbf{\Theta}} \to [0,1]$ defined by (6), $Bel(A) = \sum_{B\subset A} \frac{p(B)}{2^{n-1}}$satisfies,

1. $Bel(\phi) = 0$.
2. $Bel(\mathbf{\Theta}) = 1$.
3. Super-additive Property :

For collection $A_1, A_2, \cdots, A_n$,

$$Bel\left(\bigcup_{i=1}^{n} A_i\right) \geq \sum_{I\subset 1,2,\cdots,n, I\neq\phi} (-1)^{|I|+1} Bel\left(\bigcap_{i=1}^{n} A_i\right)$$

The basic probability assignment function defined by (6), $m(A) = \frac{p(A)}{2^{n-1}}$, for any subset $A \in \mathbf{\Theta}$, is a non-decreasing function. Also The belief function defined by basic probability assignment (6),$m(A) = \frac{p(A)}{2^{n-1}}$, for any subset $A \in \mathbf{\Theta}$, is a non-decreasing function. $Bel(\mathbf{\Theta}) = 1$, $Bel(\phi) = 0$ and if $A \neq \mathbf{\Theta}$, $A \neq \phi$ i. e. $\phi \subset A \subset \mathbf{\Theta}$, then $0 < Bel(A) < 1$. In all, $0 \leq Bel(A) \leq 1$, $\forall\, A \subseteq \mathbf{\Theta}$. The belief function defined by basic belief assignment (6), $m(A) = \frac{p(A)}{2^{n-1}}$, for any subset $A \in \mathbf{\Theta}$, is a probability measure.

Notes :

1. If $A = \mathbf{\Theta}$then$p(A) = 1$.

Therefore

$$Q(A) = \frac{1}{2^{n-1}} \sum_{B\supseteq A} p(B) = m(\mathbf{\Theta}).$$

2. If $A = \phi$ then $p(A) = 0$.

Therefore

$$Q(A) = \frac{1}{2^{n-1}} \sum_{B \supseteq A} p(B) = 1.$$

3. If $A \neq \mathbf{\Theta}$,$A \neq \phi$i. e. $\phi \subseteq A \subseteq \mathbf{\Theta}$, then

$$Q(A) = \frac{1}{2^{n-1}} \sum_{B \supseteq A} \sum_{\{b\} \in B} p(\{b\})$$

In all, $\frac{1}{2^{n-1}} \leq Q(A) \leq 1, \forall\, A \subseteq \mathbf{\Theta}$.

Theorem 3.4 A belief function $Bel: 2^{\mathbf{\Theta}} \to [0, 1]$ obtained by bba (6),$m(A) = \frac{p(A)}{2^{n-1}}$ is not Bayesian belief function.

Plausibility Function :By (6), we have, for any $A \subseteq \mathbf{\Theta}$,

$$Pl(A) = \sum_{B \cap A \neq \phi} m(B) = \frac{1}{2^{n-1}} \sum_{B \cap A \neq \phi} \sum_{\{b\} \in B} p(\{b\}) \quad (8)$$

Notes :

1. If $A = \mathbf{\Theta}$ then $p(A) = 1$.
Therefore,

$$Pl(A) = \frac{1}{2^{n-1}} \sum_{A \cap B \neq \phi} p(B) = 1$$

2. If $A = \phi$ then $p(A) = 0$.
Therefore,

$$Pl(A) = \frac{1}{2^{n-1}} \sum_{A \cap B \neq \phi} p(B) = 0$$

3. If $A \neq \mathbf{\Theta}$, $A \neq \phi$ i. e. $\phi \subset A \subset \mathbf{\Theta}$, then

$$Pl(\phi) < Pl(A) = \frac{1}{2^{n-1}} \sum_{A \cap B \neq \phi} \sum_{\{b\} \in B} p(\{b\}) < Pl(\mathbf{\Theta})$$
$$\Rightarrow 0 < Pl(A) < 1.$$

In all, $0 \leq Pl(A) \leq 1, \forall\, A \subseteq \mathbf{\Theta}$.

IV. SOME RESULTS ABOUT CALCULATING BELIEF FUNCTIONS, COMMONALITY FUNCTIONS AND PLAUSIBILITY FUNCTIONS

Theorem 4.1 If $|\mathbf{\Theta}| = n,\ 0 \leq r \leq n$ then the no. of subsets of $\mathbf{\Theta}$ containing given r elements of $\mathbf{\Theta}$ are 2^{n-r}, i.e. if$|\mathbf{\Theta}| = n$ and $A \subseteq \mathbf{\Theta}$ with $|A| = r$ then number of subsets containing A in $\mathbf{\Theta}$ are 2^{n-r}.

Proof. We have result : Any singleton set in $\mathbf{\Theta}$ with $|\mathbf{\Theta}| = n$ is repeated 2^{n-1} times in listing of all subsets of $\mathbf{\Theta}$.

This result is also true if we replace singleton sets by disjoint subsets of $\mathbf{\Theta}$.

Let $\mathbf{\Theta} = \{a_1, a_2, \cdots a_n\}$, $A = \{a_1, a_2, \cdots, a_r\}$ and $\mathbf{\Theta} = \{A = \{a_1, a_2, \cdots, a_r\}, a_{r+1}, a_{r+1}, \cdots, a_n\}$.

Hence $|\mathbf{\Theta}| = n - r + 1$.

The set $A = \{a_1, a_2, \cdots, a_n\}$ in $\mathbf{\Theta}$with $|\mathbf{\Theta}| = n - r + 1$, is repeated $2^{(n-r+1)-1} = 2^{n-r}$times in listing of all subsets of $\mathbf{\Theta}$. Hence A is also repeated 2^{n-r} times in listing of all subsets $\mathbf{\Theta}$.

Theorem 4.2 If $|\mathbf{\Theta}| = n$ and for given $A \subseteq \mathbf{\Theta}$ with $|A| = r$, then number of subsets B of $\mathbf{\Theta}$such that $A \cap B \neq \phi$ are $2^{n-r}(2^r - 1)$.

Proof. We have $|A| = r$ and $|\overline{A}| = n - r$.

Subset B with $A \cap B \neq \phi$ can be formed in following different ways as

$$\binom{r}{1}\left[\binom{n-r}{0} + \binom{n-r}{1} + \cdots + \binom{n-r}{n-r}\right] +$$

$$\binom{r}{2}\left[\binom{n-r}{0} + \binom{n-r}{1} + \cdots + \binom{n-r}{n-r}\right] +$$

$\binom{r}{3}\left[\binom{n-r}{0}+\binom{n-r}{1}+\cdots+\binom{n-r}{n-r}\right]+$

$\cdots+\cdots+$

$\binom{r}{r}\left[\binom{n-r}{0}+\binom{n-r}{1}+\cdots+\binom{n-r}{n-r}\right]$

$=\left[\binom{r}{1}+\binom{r}{2}+\cdots+\binom{r}{r}\right]\left[\binom{n-r}{0}+\binom{n-r}{1}+\cdots+\binom{n-r}{n-r}\right]$

$$= [2^r - 1][2^{n-r}] \tag{9}$$

Therefore, if $|\mathbf{\Theta}| = n$ and for given $A \subseteq \mathbf{\Theta}$ with $|A| = r$, then number of subsets B of $\mathbf{\Theta}$ such that $A \cap B \neq \phi$ are $2^{n-r}(2^r - 1)$.

Here we use the result that singletons in discrete space, are disjoint and therefore for any subsets A and B of $\mathbf{\Theta}$,$P(A \cup B) = p(A) + p(B)$.

Theorem 4.3 For all $A \subset \mathbf{\Theta}$,

$$Bel(A) = \frac{p(A)}{2^{n-k}} \text{ and } Pl(A) = p(A) + \frac{(2^k - 1)}{2^k} p(\overline{A}) \quad\quad 10)$$

where $n = |\mathbf{\Theta}|$ and $k = |A|$.

Proof. Let $n = |\mathbf{\Theta}|$ and $k = |A|$, for some $A \subset \mathbf{\Theta}$. Without loss of generality, we assume that $A = \{A_1, A_2, \cdots, A_k\}$, where $A_j, j = 1,2,\cdots,k$ are singlton sets. Let $p(A_j) = s_j$, $j = 1,2,\cdots,k$ and $p(A) = \sum_{j=1}^{k} p(A_j) = \sum_{j=1}^{k} s_j$. Therefore by (6), $m(A_j) = \frac{s_j}{2^{n-1}}$ and $m(A) = \frac{\sum_{j=1}^{k} p(A_j)}{2^{n-1}}$.

Then

$$\begin{aligned}
Bel(A) &= \sum_{B \subseteq A} m(B) \\
&= \frac{1}{2^{n-1}}\left\{\sum_{B \subseteq A} p(B)\right\} \\
&= \frac{1}{2^{n-1}}\{s_1 + s_2 + \cdots + s_k + s_1 + s_2 + s_1 + s_3 + \cdots + s_1 + s_k + s_2 + s_3 + s_2 + s_4 + \cdots \\
&\qquad + s_2 + s_k + \cdots + s_{k-1} + s_k + \cdots + s_1 + s_2 + \cdots + s_k\} \\
&= \frac{1}{2^{n-1}} \sum_{A_j \in A} p(A_j) \\
&= \frac{1}{2^{n-1}}\{2^{k-1}(s_1 + s_2 + \cdots s_k)\} \\
&= \frac{1}{2^{n-k}} P(A) \\
&= \frac{p(A)}{2^{n-k}}
\end{aligned}$$

Also,

$$\begin{aligned}
Pl(A) &= 1 - Bel(\overline{A}) \\
&= 1 - \frac{p(\overline{A})}{2^{n-(n-k)}} \\
&= \frac{2^k - p(\overline{A})}{2^k}
\end{aligned}$$

$$= \frac{2^k\left(p(A) - p(\overline{A})\right) - p(\overline{A})}{2^k}$$
$$= p(A) + \frac{(2^k - 1)}{2^k} p(\overline{A})$$

Notes :

1. By above theorem, we have $Bel(A) \leq p(A) \leq Pl(A), \forall A \subseteq \mathbf{\Theta}$.
2. If Bel is a Bayesian belief function then the function p is given by $p(\theta) = 2^{n-1} m(\theta)$.

Theorem 4.4 For any $A \subset \mathbf{\Theta}$ with $|\mathbf{\Theta}| = n, |A| = r$ then

$$Q(A) = 2^{1-r} \sum_{\{a\} \in A} p(\{a\}) + \frac{1}{2^r} \sum_{\{a\} \notin A} p(\{a\}) \tag{11}$$

Proof. We have result: If $|\mathbf{\Theta}| = n$ and $A \subseteq \mathbf{\Theta}$ with $|A| = r$ then number of subsets containing $A \subseteq \mathbf{\Theta}$ are 2^{n-r}. Hence subset A is repeated 2^{n-r} times in calculation of commonality of A. Therefore any element in A is repeated 2^{n-r} times in calculation of commonality of A.

The element $\{a\} \notin A$ also have contribution in calculation of commonality of A. Such element $\{a\} \notin A$ is repeated 2^{n-r-1} times in listing of super sets of $\{a\}$ in $\overline{A}$, for calculation of commonality of A. By definition of commonality,

$$\begin{aligned}
Q(A) &= \sum_{B \supseteq A} m(B) \\
&= \sum_{B \supseteq A} \frac{p(A)}{2^{n-1}} \\
&= \frac{1}{2^{n-1}} [2^{n-r} \sum_{\{a\} \in A \supseteq B} p(\{a\}) + 2^{n-r-1} \sum_{\{a\} \notin A, \{a\} \in B} p(\{a\})] \\
&= \frac{1}{2^{n-1}} \left[2^{n-r} \sum_{\{a\} \in A} p(\{a\}) + 2^{n-r-1} \sum_{\{a\} \notin A, \{a\} \in B} p(\{a\})\right] \\
&= 2^{1-r} \sum_{\{a\} \in A} p(\{a\}) + 2^{-r} \sum_{\{a\} \in A, \{a\} \in B} p(\{a\}) \\
&= \frac{1}{2^{r-1}} \sum_{\{a\} \in A} p(\{a\}) + \frac{1}{2^r} \sum_{\{a\} \in A, \{a\} \in B} p(\{a\})
\end{aligned}$$

By using theorems 4.1 and 4.2, we have alternate proof of part of theorem 4.3.

Theorem 4.5 For any subset A of $\mathbf{\Theta}$with $|\mathbf{\Theta}| = n$ and $|A| = r$,

$$Pl(A) = \sum_{\{a\} \in A} p(\{a\}) + \sum_{\{a\} \notin A} \left(1 - \frac{1}{2^r}\right) p(\{a\}) \text{ i.e. } Pl(A) = p(A) + \left(\frac{2^r - 1}{2^r}\right) p(\overline{A}) \tag{12}$$

Proof. In general, observing carefully, we notice that number of repetitions of element of $\mathbf{\Theta}$ for plausibility is as follows:

Let $|A| = r$ and $|\mathbf{\Theta}| = n$ if $\{a\} \in A$ then $\{a\}$ appears 2^{n-1} times and if $\{a\} \notin A$ then $\{a\}$ appears $2^{n-2} + 2^{n-3} + 2^{n-4} + \cdots + 2^{n-(r+1)}$ times, hence $2^{n-(r+1)}(2^r - 1)$ times. Therefore formula for plausibility function becomes

$$\begin{aligned}
Pl(A) &= \sum_{A \cap B \neq \phi} m(B) \\
&= 2^{n-1} \sum_{\{a\} \in A} \frac{p(\{a\})}{2^{n-1}} + 2^{n-(r+1)}(2^r - 1) \sum_{\{a\} \notin A} \frac{p(\{a\})}{2^{n-1}} \\
&= \sum_{\{a\} \in A} p(\{a\}) + 2^{-r}(2^r - 1) \sum_{\{a\} \notin A} p(\{a\}) \\
&= \sum_{\{a\} \in A} p(\{a\}) + \left(1 - \frac{1}{2^r}\right) \sum_{\{a\} \notin A} p(\{a\}) \\
&= p(A) + \left(1 - \frac{1}{2^r}\right) p(\overline{A})
\end{aligned}$$

V. CONCLUSION

In this paper, we employ a novel basic belief assignment derived from the probability mass function of a discrete probability distribution. Through this approach, we explore various theorems related to belief and plausibility functions.

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