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Outer-restrained Domination in the Join and Corona of Graphs

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Abstract: Let G be a connected simple graph. A set $S \subseteq V(G)$ is a restrained dominating set if every vertex not in S is adjacent to a vertex in S and to a vertex in $V(G) \setminus S$. A set S of vertices of a graph G is an outer-restrained dominating set if every vertex not in S is adjacent to some vertex in S and $V(G) \setminus S$ is a restrained set. The outer-restrained domination number of G, denoted by $\widetilde{\gamma_r}(G)$ is the minimum cardinality of an outer-restrained dominating set of G. An outer-restrained set of cardinality $\widetilde{\gamma_r}(G)$ will be called $\widetilde{\gamma_r}$- set. In this paper, we initiate a study of the concept and give the characterization of the outer-restrained dominating set in the join and corona of two graphs.

Keywords: corona, domination, join, outer, restrained

I. INTRODUCTION

Let G be a connected simple graph. A set S of vertices of G is a dominating set of G if every vertex in $V(G) \setminus S$ is adjacent to some vertex in S. A minimum dominating set in a graph G is a dominating set of minimum cardinality. The cardinality of a minimum dominating set in G is called the *domination number* of G and is denoted by $\gamma(G)$. The concept of domination in graphs introduced by Claude Bergein 1958 and Oystein Orein 1962 [1] is currently receiving much attention in literature. Following the article of Ernie Cockayne and Stephen Hedetniemi [2], the domination in graphs became an area of study by many researchers [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

Other type of domination parameter is the restrained domination numberin a graph.A*restrained dominating set* is defined to be a set $S \subseteq V(G)$where everyvertex in $V(G) \setminus S$ is adjacent to a vertex in $S$and to another vertex in $V(G) \setminus S$. The *restrained domination number* of G, denoted by $\gamma_r(G)$, is the smallest cardinality of a restrained dominating set of G. This was introduced by Telle and Proskurowski [18] indirectly as a vertex partitioning problem. Note that $S = V(G)$ is restrained dominating set and the decision problem for $\gamma_r(G)$ is an NP-complete [19]. Some studies on restrained domination in graphs can be found in [20, 21, 22, 29, 24, 25,26, 27, 28, 29, 30].

A graph G is *connected* if there is at least one path that connects every two vertices $x, y \in V(G)$, otherwise, G is *disconnected*. A set S of vertices of a graph G is an *outer-connected dominating set* if every vertex not in S is adjacent to some vertex in S and the sub-graph induced by $V(G) \setminus S$ is connected. The *outer-connected domination number*$\widetilde{\gamma_c}(G)$ is the minimum cardinality of the outer-connected dominating set S of a graph G. The concept of outer-connected domination in graphs was introduced by Cyman [31]. Variant of outer-connected domination in graphs can be found in [32, 33, 34, 35].

A graph G is a pair $V(G), E(G)$, where $V(G)$ is a nonempty finite set whose elements are called *vertices* and $E(G)$ is a set of unordered pairs of distinct elements of $V(G)$. The elements of $E(G)$ are called *edges* of the graph G. The number of vertices in G is called the *order* of G and the number of *edges* is called the size of G. For more graph-theoretical concepts, the readers may refer [36].

Motivated by the concepts of restrained domination, and outer-connected domination in graphs, the researchers introduced a new variant of domination in graphs, the *outer-restrained domination* in graphs. A set S of vertices of a graph G is an *outer-restrained dominating set* if every vertex not in S is adjacent to some vertex in S and $V(G) \setminus S$ is a restrained set. The *outer-restrained domination number* of G, denoted by $\widetilde{\gamma_r}(G)$is the minimum cardinality of an outer-restrained dominating set of G. An outer-restrained set of cardinality $\widetilde{\gamma_r}(G)$will be called $\widetilde{\gamma_r}$*-set.* In this paper, we initiate a study of the concept and give the characterization of the outer-restrained dominating set in the join and corona of two graphs.

II. RESULTS

The graphs G considered here are simple, finite, nontrivial, undirected and without isolated vertices.

Definition 2.1 *A set S of vertices of a graph G is an outer-restrained dominating set if every vertex not in S is adjacent to some vertex in S and $V(G) \setminus S$ is a restrained set. The outer-restrained domination number of G, denoted by $\widetilde{\gamma_r}(G)$ is the minimum cardinality of an outer-restrained dominating set of G. An outer-restrained set of cardinality $\widetilde{\gamma_r}(G)$ will be called $\widetilde{\gamma_r}$-set.*

Definition 2.2 *The join $G + H$ of two graphs G and H is the graph with vertex set $V(G+H) = V(G) \cup V(H)$ and edge-set $E(G+H) = E(G) \cup E(H) \cup \{uv : u \in V(G), v \in V(H)\}$.*

Remark 2.3 *Let G and H be nontrivial connected graphs. Then*

$$\widetilde{\gamma_r}(G+H) \neq 1.$$

Lemma 2.4 *Let G and H be nontrivial connected graphs. If S is a dominating set of G and for every $v \in S$ there exists another $v' \in S$ such that $vv' \in E(G)$, then a nonempty $S \subseteq V(G+H)$ is an outer-restrained dominating set of $G+H$.*

Proof: If S is a dominating set of G, then S is a dominating set of $(G+H)$. Thus, every vertex $u \in V(G+H)\setminus S$, there exists $v \in S$ such that $uv \in E(G+H)$. Furthermore, for every $v \in S = V(G+H)(V(G+H)\setminus S)$, there exists another $v' \in S = V(G+H)\setminus(V(G+H)\setminus S)$ such that $vv' \in E(G+H)$. This implies that for every $u \in V(G+H)\setminus S$, there exists $v \in V(G+H)\setminus V(G+H)\setminus S)$ such that $uv \in E(G+H)$, and there exists another $v' \in V(G+H)\setminus V(G+H)\setminus S)$ such that $vv' \in V(G+H)$, that is, $V(G+H)\setminus S$ is a restrained set. Thus, for every vertex not in S is adjacent to some vertex in S and $V(G+H)\setminus S$ is a restrained set. By Definition 2.1, a nonempty set $S \subseteq V(G+H)$ is an outer-restrained dominating set of $G+H$. ■

Lemma 2.5 *Let G and H be a nontrivial connected graphs. If S is a dominating set of H and for every $v \in S$, there exists another $v' \in S$ such that $vv' \in E(H)$, then a nonempty $S \subseteq V(G+H)$ is an outer-restrained dominating set of $G+H$.*

Proof: Suppose S is a dominating set of H and for every $v \in S$, there exists another $v' \in S$ such that $vv' \in E(H)$. This proof is similar to the proof of Lemma 2.4. ■

Lemma 2.6 *Let G and H be a nontrivial connected graphs. If $S = S_G \cup S_H$ where $S_G \subseteq V(G)$ and $S_H \subseteq V(H)$, then a nonempty $S \subseteq V(G+H)$ is an outer-restrained dominating set of $G+H$.*

Proof: Suppose that $S = S_G \cup S_H$ where $S_G \subseteq V(G)$ and $S_H \subseteq V(H)$.
Consider the following cases.

Case 1. If $S_G = V_G$ and $S_H = V(H)$, then $S = V(G+H)$ is a dominating set and $V(G+H)\setminus S$ is a restrained set (vacuously satisfy the desired property). Hence, a nonempty $S \subseteq V(G+H)$ is an outer-restrained dominating set of $G+H$.

Case 2. If $S_G = V(G)$ and $S_H \neq V(H)$, then $S = V(G) \cup S_H$ is a dominating set of $G+H$ and $V(G+H)\setminus S = V(H)\setminus S_H$ is a restrained set because every v not in $V(H)\setminus S_H$, there exists $u \in V(H)\setminus S_H$ such that $vu \in E(G+H)$ and there exists another v' not in $V(H)\setminus S_H$ such that $vv' \in E(G+H)$. Hence, a nonempty $S \subseteq V(G+H)$ is an outer-restrained dominating set of $G+H$.

Case 3. If $S_G \neq V(G)$ and $S_H = V(H)$, then $S = S_G \cup V(H)$ is dominating set of $G+H$ and $V(G+H)\setminus S = V(G)\setminus S_G$ is a restrained set because every v not in $V(G)\setminus S_G$, there exists $u \in V(G)\setminus S_G$ such that $vu \in E(G+H)$ and there exists another v' not in $V(G)\setminus S_G$ such that $vv' \in E(G+H)$. Hence, a nonempty $S \subseteq V(G+H)$ is an outer-restrained dominating set of $G+H$.

Case 4. If $S_G \neq V(G)$ and $S_H \neq V(H)$, then $S = S_G \cup S_H$ is dominating set of $G+H$ and $V(G+H)\setminus S = (V(G)\setminus S_G) \cup (V(H)\setminus S_H)$ is restrained set because every v not in $V(G+H)\setminus S$, there exists $u \in V(G+H)\setminus$

Ssuch that $vu \epsilon E(G+H)$and there exists another v'not in $V(G+H)\backslash S$ such that $vv' \epsilon E(G+H)$. Hence, a nonempty $S \subseteq V(G+H)$ is an outer-restrained dominating set of $G+H$.■

The following result is the characterization of an outer-restrained dominating set in the join of two graphs.

Theorem 2.7*Let G and H be a nontrivial connected graphs. Then a nonempty $S \subseteq V(G+H)$is an outer-restrained dominating set of $G+H$ if and only if one of the following is satisfied:*

i. *S is a dominating set of G and for every v ∈S, there exists another v' ∈S such that$vv' \in E(G)$.*
ii. *$S is a dominating set of H and for every v \in S, there exists another v' \in S$ such that$vv' \in E(H)$.*
iii. *$S = S_G \cup S_H$ where$S_G \subseteq V(G)$and$S_H \subseteq V(H)$.*

Proof: Suppose that a nonempty $S \subseteq V(G+H)$is an outer-restrained dominating set of $G+H$. Then every vertex not in *S* is adjacent to some vertex in *S* and $V(G+H)\backslash S$is a restrained set.

Case 1. If $S \cap V(G) = \phi$, then $S \subseteq V(G)$. This implies that *S* is a dominating set of *G* (since *S* is a dominating set of *G* + *H*) and $V(G)\backslash S$ is a restrained set. Thus every $v \in V(G)\backslash(V(G)\backslash S) = S$,there exists $u \in V(G)\backslash S$ such that $vu \in E(G)$and another vertex in $v' \in V(G)\backslash(V(G)\backslash S) = S$ such that $vv' \in E(G)$. This shows statement (*i*).

*Case 2.*If $S \cap V(G) = \phi$, then $S \subseteq V(H)$. This implies that*S* is a dominating set of *H* (since *S* is dominating set of *G* + *H*) and $V(H)\backslash S$ is a restrained set. Thus every $v \in V(H)\backslash(V(H)\backslash S) = S$, there exists $u \in V(H)\backslash S$ such that $vu \in E(H)$and another vertex in $v' \in V(H)\backslash(V(H)\backslash S) = S$ such that $vv' \in E(H)$. This shows statement (*ii*).

Case 3. If$S \cap V(G) \neq \phi$ and $S \cap V(H) \neq \phi$, thenlet $S_G = S \cap V(G)$and$S_H = S \cap V(H)$. This implies that

$$\begin{aligned} S_G \cup S_H &= (S \cap V(G)) \cup (S \cap V(H)) \\ &= S \cap (V(G) \cup V(H)) \\ &= S \cap V(G+H) \\ &= S, \text{ that is,} \end{aligned}$$

$S = S_G \cup S_H$ where$S_G \subseteq V(G)$and$S_H \subseteq V(H)$. This shows statement (*iii*).

For the converse, suppose that statement (*i*) is satisfied. Then *S* is a dominating set of *G*and for every *v*∈*S*, there exists another$v' \in S$such that$vv' \in E(G)$.By Lemma 2.4, a nonempty$S \subseteq V(G+H)$ is an outer-restrained dominating set of $G+H$.

Suppose that statement (*ii*) is satisfied. Then *S* is a dominating set of *H* and for every *v*∈*S*, there exists another$v' \in S$such that $vv' \in E(H)$. By Lemma 2.5, a nonempty$S \subseteq V(G+H)$ is an outer-restrained dominating set of $G+H$.

Suppose that statement (*iii*) is satisfied. Then $S = S_G \cup S_H$ where $S_G \subseteq V(G)$and$S_H \subseteq V(H)$. By Lemma 2.6, a nonempty $S \subseteq V(G+H)$is an outer-restrained dominating set of $G+H$.■

The following result is an immediate consequence of Theorem 2.7.

Corollary 2.8*Let G and H be nontrivial connected graphs. Then*

$$\tilde{\gamma}_r(G+H) = 2.$$

*Proof:*Let $S = \{x, y\}$where$x \in V(G)$and$y \in V(H)$. Then$S = S_G \cup S_H$ where$S_G = \{x\}$and$S_H = \{y\}$. By Theorem 2.7, *S* is an outer-restrained dominating set of $G+H$. Thus, $\tilde{\gamma}_r(G+H) \leq |S| = 2$. By Remark 2.3, $\tilde{\gamma}_r(G+H) \leq |S| \neq 1$,that is, $\tilde{\gamma}_r(G+H) \geq 2$. This implies that $2 \leq \tilde{\gamma}_r(G+H) \leq 2$. Hence, $\tilde{\gamma}_r(G+H) = 2$. ■

Definition 2.9*Let G and H be graphs of order m and n, respectively. The corona of G and H, denotedby$G \circ H$, is the graph obtained by taking one copy of G and m copiesof H, and then joining the ith vertex of G to every vertex of the ith copy of H. The join of vertex v of G and a copy of H^v of H in the corona of G and H is denoted by$v + H^V$.*

Let *G* be a connected graph and $x \in V(G)$. Since $V(G)\backslash\{x\}$ is not a dominating set of $G \circ H$ for any simple graph *H*, the following remark holds.

Remark 2.10*Let G be a connected graph and H be any graph. Then $V(G)$ is a minimum dominating set of* $G \circ H$.

The following results is the characterization of an outer-restrained dominating set in the corona of two graphs.

Theorem 2.11*Let G and H be nontrivial connected graphs. Then a nonempty $S \subseteq V(G \circ H)$ is an outer-restrained dominating set of $G \circ H$ if and only if one of the following is satisfied.*

i. $S = V(G) \cup \left(\bigcup_{v \in V(G)} S_v\right)$ *where* $\emptyset \subseteq S_v \subseteq V(H^v)$ *for all* $v \in V(G)$

ii. $S = \bigcup_{v \in V(G)} (S_v)$ *where S_v is a connected dominating set of H^v with $|S_v| \geq 2$ for all $v \in V(G)$.*

Proof: Suppose that a nonempty $S \subseteq V(G \circ H)$ is an outer-restrained dominating set of $G \circ H$.

If $S = V(G \circ H)$, then

$$S = V(G) \cup \left(\bigcup_{v \in V(G)} S_v\right)$$

where $S_v = V(H^v)$ for all $v \in V(G)$. Suppose that $S \neq V(G \circ H)$, that is, $S \subset V(G \circ H)$.

Consider the following cases.

Case 1. If $V(G) \neq \emptyset$, then

$$S = V(G) \cup \left(\bigcup_{v \in V(G)} S_v\right)$$

where $\emptyset \subseteq S_v \subset V(H^v)$ for all $v \in V(G)$.

Case 2. If $V(G) \neq \emptyset$, then

$$S = \bigcup_{v \in V(G)} (S_v)$$

where $\emptyset \subset S_v \subset V(H^v)$ for all $v \in V(G)$(since S is nonempty).

Suppose S_v is not a dominating set of H^v for each $v \in V(G)$. Then S is not a dominating set contrary to the assumption that S is a dominating set of $G \circ H$. Thus, S_v must be a dominating set of H^v for each $v \in V(G)$. Suppose that S_v is not connected for each $v \in V(G)$. Then S is not connected and $V(G \circ H) \backslash S$ is not a restrained set, contrary to the definition of *S*. Thus, S_v is connected for each $v \in V(G)$. Clearly, $|S_v| \geq 2$, otherwise, *S* is not connected.

Therefore, if $S \subseteq V(G \circ H)$ and $V(G) \neq \emptyset$, then

$$S = V(G) \cup \left(\bigcup_{v \in V(G)} S_v\right)$$

where $\emptyset \subseteq S_v \subseteq V(H^v)$ for all $v \in V(G)$, satisfying statement (i).

If $S \subset V(G \circ H)$ and $V(G) \neq \emptyset$, then

$$S = \bigcup_{v \in V(G)} S_v$$

where S_v is a connected dominating set of H^v with $|S_v| \geq 2$, for all $v \in V(G)$, satisfying statement (ii).

For the converse, suppose that statement (i) is satisfied. Then

$$S = V(G) \cup \left(\bigcup_{v \in V(G)} S_v \right)$$

where $\emptyset \subseteq S_v \subseteq V(H^v)$ for all $v \in V(G)$.

Consider the following cases.

Case 1. If $S_v = V(H^v)$ for all $v \in V(G)$, then

$$S = V(G) \cup \left(\bigcup_{v \in V(G)} V(H^v) \right)$$

= $V(G \circ H)$ is an outer-restrained dominating set (vacuously satisfy the desired property).

Case 2.If $S_v \neq V(H^v)$ for each $v \in V(G)$, then consider the following.

Subcase 1. If $S_v \neq \emptyset$, then

$$S = V(G) \cup \left(\bigcup_{v \in V(G)} \emptyset \right) = V(G)$$

Then S is a dominating set of $G \circ H$ by Remark 2.10.

Further,

$$D = V(G \circ H) \setminus S = \bigcup_{v \in V(G)} V(H^v)$$

is a restrained set of $G \circ H$because for every $v \in V(G \circ H) \setminus D = S$, there exists $u \in D$ such that $vu \in E(G \circ H)$and there exists another $v' \in S$ such that $vv' \in E(G \circ H)$.Hence, a nonempty $S \subseteq V(G \circ H)$ is an outer-restrained dominating set of $G \circ H$.

Subcase 2. If $S_v \neq \emptyset$, then

$$S = V(G) \cup \left(\bigcup_{v \in V(G)} S_v \right) \emptyset \subset S_v \subset V(H^v)$$

for all $v \in V(G)$. Since $V(G) \subset S$, it follows that S is a dominating set of $G \circ H$.

Now,

$$D = \bigcup_{v \in V(G)} (V(H^v) \setminus S_v)$$

is a restrained set for any $S_v \subset V(H^v)$, for all $v \in V(G)$because for some $v \in V(G \circ H) \setminus D = S$, there exists $u \in D$ such that $vu \in E(G \circ H)$ and there exists another $v' \in S$ such that $vv' \in E(G \circ H)$. Hence, a nonempty $S \subseteq V(G \circ H)$ is an outer-restrained dominating set of $G \circ H$.

Suppose that statement (ii) is satisfied. Then

$$S = \bigcup_{v \in V(G)} S_v$$

where S_v is a connected dominating set of H^v with $|S_v| \geq 2$ for all $v \in V(G)$. Since S_v is a dominating set of H^v for each $v \in V(G)$, it follows that S is a dominating set of $G \circ H$.

Now

$$D = \bigcup_{v \in V(G)} (V(H^v) \setminus S_v)$$

is a restrained set of $G \circ H$ for all $v \in V(G)$ because for every $v \in V(G \circ H) \setminus D = S$, there exists $u \in D$ such that $vu \in E(G \circ H)$ and there exists another $v' \in S$ (note that $|S_v| \geq 2$ for all $v \in V(G)$such that $vv' \in E(G \circ H)$. Hence, a nonempty $S \subseteq V(G \circ H)$ is an outer-restrained dominating set of $G \circ H$. ■

The following result is an immediate consequence of Theorem 2.11.

Corollary 2.12*Let G and H be nontrivial connected graphs. Then*

$$\widetilde{\gamma_r}(G \circ H) = |V(G)|.$$

*Proof:*Suppose that $S = V(G)$. In view of Theorem2.11 (i) if $S_v \neq \emptyset$, then $S$is an outer-restrained dominating set of $G \circ H$. Thus, $\widetilde{\gamma_r}(G \circ H) \leq |S| = |V(G)|$.

In Remark 2.10, the $\widetilde{\gamma_r}(G \circ H) = |V(G)|$. Thus,

$$|V(G)| = \gamma(G \circ H) \leq \widetilde{\gamma_r}(G \circ H) \leq |S| = |V(G)| .$$

Therefore, $\widetilde{\gamma_r}(G \circ H) = |V(G)|$. ■

III. CONCLUSION AND RECOMMENDATIONS

In this paper, we introduced a new parameter of domination in graphs - the outer-restrained domination in graphs. The outer-restrained domination in the join and corona of two graphs were characterized and the exact value of outer-restrained domination number resulting from the join and corona of two graphs were computed. This study will pave the way to new researches such as bounds and other binary operations of two connected graphs. Other parameters involving outer-restrained domination in graphs may also be explored. Finally, the characterization of an outer-restrained domination in graphs of the lexicographic and Cartesian products of two graphs, and its bounds are promising extension of this study.

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REFERENCES

[1]. O. Ore. Theory of Graphs. American Mathematical Society, Provedence, R.I., 1962.

[2]. E.J. Cockayne, and S.T. Hedetniemi. Towards a theory of domination in graphs, Networks, (1977) 247-261.

[3]. E.L. Enriquez, and S.R. Canoy, Jr. On a variant of convex domination in a graph, International Journal of Mathematical Analysis, 9 (32), *1585-1592.*http://dx.doi.org/10.12988/ajma.2015.54127.

[4]. E.L. Enriquez, and S.R. Canoy, Jr., Secure Convex Domination in a Graph, International Journal of Mathematical Analysis, Vol. 9, 2015, *no. 7, 317-325.*

[5]. Grace M Estrada, Carmelita M Loquias, Enrico L Enriquez, Christine S Baraca, Perfect doubly connected domination in the join and *corona of graphs, International Journal of Latest Engineering Research and Applications, 4(7), 2019, pp 11-16.*

[6]. Enrico Enriquez, Valerie Fernandez, Teodora Punzalan, JonecisDayap, Perfect outer-connected domination in the join and corona of *graphs, Recoletos Multidisciplinary Research Journal, 4(2), 2016.*

[7]. M.D. Garol, E.L. Enriquez, K.E. Belleza, G.M. Estrada, C.M. Loquias, Disjoint Fair Domination in the Join and Corona of Two *Graphs, International Journal of Mathematics Trends and Technology, 68(2), 2022, pp 124-132.*

[8]. Jonecis Dayap, Enrico Enriquez, Disjoint secure domination in the join of graphs, Recoletos Multidisciplinary Research Journal, 4(2), *2016.*

[9]. N.A. Goles, E.L. Enriquez, C.M. Loquias, G.M. Estrada, R.C. Alota, z-Domination in Graphs, Journal of Global Research in Mathematical*Archives, 5(11), 2018, pp 7-12.*
[10]. E.L. Enriquez, V.V. Fernandez, J.N. Ravina, Outer-clique Domination in the Corona and Cartesian Product of Graphs, Journal of Global *Research in Mathematical Archives, 5(8), 2018, pp 1-7.*
[11]. E.L. Enriquez, G.M. Estrada, V.V. Fernandez, C.M. Loquias, A.D. Ngujo, Clique Doubly Connected Domination in the Corona and*Cartesian Product of Graphs, Journal of Global Research in Mathematical Archives, 6(9), 2019, pp 1-5.*
[12]. E.L. Enriquez, G.M. Estrada, C.M. Loquias, Weakly Convex Doubly Connected Domination in the Join and Corona of Graphs, Journal*of Global Research in Mathematical Archives, 5(6), 2018, pp 1-6.*
[13]. J.A. Dayap, E.L. Enriquez, Outer-convex Domination in Graphs in the Composition and Cartesian Product of Graphs, Journal of Global *Research in Mathematical Archives, 6(3), 2019, pp 34-42.*
[14]. D.P. Salve, E.L. Enriquez, Inverse Perfect Domination in the Composition and Cartesian Product of Graphs, Global Journal of Pure and *Applied Mathematics, 12(1), 2016, pp 1-10.*
[15]. E.L. Enriquez, B.P. Fedellaga, C.M. Loquias, G.M. Estrada, M.L. Baterna, Super Connected Domination in Graphs, Journal of Global *Research in Mathematical Archives, 6(8), 2019, pp 1-7.*
[16]. E.L. Enriquez, E.S. Enriquez, Convex Secure Domination in the Join and Cartesian Product of Graphs, Journal of Global Research in *Mathematical Archives, 6(5), 2019, pp 1-7*
[17]. M.P. Baldado, Jr. and E.L. Enriquez, Super Secure Domination in Graphs, International Journal of Mathematical Archive-8(12), 2017*, pp. 145-149.*
[18]. J.A. Telle, A. Proskurowski, Algorithms for Vertex Partitioning Problems on Partial-k Trees, SIAM J. Discrete Mathematics, 10(1997), *529-550.*
[19]. G.S. Domke, J.H. Hattingh, S.T. Hedetniemi, R.C. Laskar, L.R. Markus, Restrained domination in graphs, Discrete Mathematics*203(1999), 61-69.*
[20]. E.L. Enriquez, On Restrained Clique Domination in Graphs, Journal of Global Research in Mathematical Archives, Vol. 4, 2017, no. *12, 73-77.*
[21]. E.L. Enriquez, Super Restrained Domination in the Corona of Graphs, International Journal of Latest Engineering Research and *Applications, Vol. 3, 2018, no. 5, 1-6.*
[22]. E.L. Enriquez, and S.R. Canoy,Jr.,Restrained Convex Dominating Sets in the Corona and the Products of Graphs, Applied Mathematical *Sciences, Vol. 9, 2015, no. 78, 3867-3873.*
[23]. [23] E.L. Enriquez, Secure Restrained Convex Domination in Graphs, International Journal of Mathematical Archive, Vol. 8, 2017, no.7,1-*5.*
[24]. DH. P. Galleros, E.L. Enriquez, Fair Restrained Dominating Set in the Cartesian Product and Lexicographic Product of Graphs,*International Journal of Mathematics Trends and Technology, 67(7), 2021, pp 87-93.*
[25]. R.C. Alota, and E.L. Enriquez, On Disjoint Restrained Domination in Graphs, Global Journal of Pure and Applied Mathematics, 12, No. *3(2016), pp. 2385-2394.*
[26]. SP.G. Cajigas, E.L. Enriquez, K.E. Belleza, G.M. Estrada, C.M.Loquias, Disjoint Restrained Domination in the Join and Corona of*Graphs, International Journal of Mathematics Trends and Technology, 67(12), 2021, pp 57-61.*
[27]. T.J. Punzalan, and E.L. Enriquez, Inverse Restrained Domination in Graphs, Global Journal of Pure and Applied Mathematics, Vol. 3,*2016, pp 1-6.*
[28]. C.M. Loquias, and E.L. Enriquez, On Secure Convex and Restrained Convex Domination in Graphs, International Journal of Applied*Engineering Research, Vol. 11, 2016, no. 7, 4707-4710.*
[29]. E.L. Enriquez, Secure Restrained Convex Domination in Graphs, International Journal of Mathematical Archive, Vol. 8, 2017, no.7,15.
[30]. E.M. Kiunisala, and E.L. Enriquez, "Inverse Secure Restrained Domination in the Join and Corona of Graphs, International Journal of*Applied Engineering Research, Vol. 11, 2016, no. 9, 6676- 6679.*
[31]. J. Cyman, The Outer-connected Domination Number of a graph, Australas. J. Combin., 38(2007), 35-46.
[32]. *E.M. Kiunisala, and E.L. Enriquez, Clique Secure Domination in Graphs, Global Journal of Pure and Applied Mathematics. Vol. 12,No. 3 (2016), pp. 2075-2084.*
[33]. C. M. Loquias, E. L. Enriquez, and J. Dayap. Inverse Clique Domination in Graphs, Recoletos Multidisciplinary Research Journal. *Vol. 4, No. 2 (2017), pp 23-34.*
[34]. R.A. Bacalso and E.L. Enriquez, On Restrained Clique Domination in Graphs, Journal of Global Research in Mathematical Archives,*Vol. 4, 2017, no. 12, 73-77.*
[35]. J.N. Ravina, Valerie V. Fernandez, and E.L. Enriquez, Outer-clique Domination in Graphs, Journal of Global Research in *Mathematical Archives, Vol. 5, 2018, no. 7.*
[36]. G. Chartrand and P. Zhang. A First Course in Graph Theory, Dover Publication, Inc., New York, 2012.

