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Fair Inverse Domination in the Corona of Two Connected Graphs

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Abstract: Let G be a nontrivial connected simple graph. A subset S of V(G) is a dominating set of G if for every v ∈ V(G)\S, there exists x ∈ Ssuch that xv ∈ E(G). Let D be a minimum dominating set of G. If V(G)\D contains a dominating set say S of G, then S is called an inverse dominating set with respect to D. A fair dominating set in a graph G (or FD-set) is a dominating set S such that all vertices not in S are dominated by the same number of vertices from S ; that is, every two vertices not in S has the same number of neighbors in S. An inverse dominating subset S of a vertex set V(G) is said to be a fair inverse dominating set if for every vertex v ∈ V(G)\Sis dominated by the same number of the vertex in S. A fair inverse domination number is the minimum cardinality of a fair inverse dominating set S in G, denoted by$\gamma_{fd}^{(-1)}(G)$. In this paper, we initiate the study of the concept and give the characterization of the fair inverse dominating set in the corona of two nontrivial connected graphs. Further, the domination number of the corona of two graphs was characterized.

Keywords: dominating set, inverse dominating set, fair dominating set, fair inverse dominating set, corona

1. Introduction

Domination in graphs was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [1]. Following an article [2] by Ernie Cockayne and Stephen Hedetniemi in 1977, the domination in graphs became an area of study bymany researchers. A subset S of $V(G)$ is a *dominating set* of G if for every $v \in V(G)\backslash S$, there exists $x \in S$ such that $xv \in E(G)$, i.e., $N[S] = V(G)$. The *domination number* $\gamma(G)$ of G is the smallest cardinality of a dominating set of G. Some studies on domination in graphs were found in the papers [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

Let D be a minimum dominating set in G. The dominating set $S \subseteq V(G)\backslash D$ is called an inverse dominating set with respect to D. The minimum cardinality of an inverse dominating set is called an inverse domination number of G and is denoted by $\gamma^{-1}(G)$. The concept of inverse domination in graphs is found in [15]. Some related studies of inverse domination in graphs are found in [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26].

Other variant of domination in a graph is the fair domination in graphs [27]. A dominating subset S of $V(G)$ is a fair dominating set of G if all the vertices are not in S are dominated by the same number of vertices from S, that is, $|N(u) \cap S| = |N(v) \cap S|$ for every two distinct vertices u and v from $V(G) \setminus S$ and a subset S of $V(G)$ is a k-fair dominating set in G if for every vertex $v \in V(G) \setminus S$, $|N(v) \cap S| = k$. The minimum cardinality of a fair dominating set of G, denoted by $\gamma_{fd}(G)$, is called the fair domination number of G. A fair dominating set of cardinality$\gamma_{fd}(G)$ is called γ_{fd}-set. Some related studies of fair domination in graphs are found in [28, 29, 30, 31, 32, 33, 34, 35].

Motivated by the introduction of the fair dominating sets and the inverse dominating sets, a new variant of domination in graphs is introduced in this paper. An inverse dominating subset S with respect to a minimum dominating set of a vertex set $V(G)$ is said to be a fair inverse dominating set if for every vertex $v \in V(G)\setminus S$ is dominated by the same number of the vertex in S. A fair inverse domination number is the minimum cardinality of a fair inverse dominating set S in G, denoted by $\gamma_{fd}^{(-1)}(G)$. In this paper, we initiate the study of the concept and give the characterization of the fair inverse dominating set in the corona of two nontrivial connected graphs. Further, the domination number of the corona of two graphs was characterized.

For the general terminology in graph theory, readers may refer to [36]. A *graphG* is a pair $(V(G), E(G))$, where $V(G)$ is a finite nonempty set called the *vertex-set* of G and $E(G)$ is a set of unordered pairs $\{u, v\}$ (or simply uv) of distinct elements from $V(G)$ called the *edge-set* of G. The elements of $V(G)$ are called *vertices* and the cardinality $|V(G)|$ of $V(G)$ is the *order* of G. The elements of $E(G)$ are called *edges* and the cardinality $|E(G)|$ of $E(G)$ is the *size* of G. If $|V(G)| = 1$, then G is called a *trivial graph*. If $E(G) = \emptyset$, then G is called an *empty graph*. The *open neighborhood* of a vertex $v \in V(G)$ is the set $N_G(v) = \{u \in V(G) : uv \in E(G)\}$. The elements of $N_G(v)$ are called *neighbors* of v. The *closed neighborhood* of $v \in V(G)$ is the set $N_G[v] = N_G(v) \cup \{v\}$. If $X \subseteq V(G)$, the open neighborhood of X in G is the set $N_G(X) = \bigcup_{v \in X} N_G(v)$.The *closed neighborhood* of X in G is the set $N_G[X] = \bigcup_{v \in X} N_G[v] = N_G(X) \cup X$. When no confusion arises, $N_G[x]$ [resp. $N_G(x)$] will be denoted by $N[x]$ [resp. $N(x)$].

2. Results

Definition 2.1 A simple graph G is an undirected graph with no loop edges or multiple edges.

Definition 2.2 The path $P_n = \{a_1a_2a_3 \ldots a_n\}$ is the graph with $V(P_n) = \{a_1, a_2, a_3, \ldots, a_n\}$ and $E(P_n) = \{a_1a_2, a_2a_3, \ldots, a_{n-1}a_n\}$.

Definition 2.3 The cycle C_n= $\{a_1a_2a_3 \ldots a_na_1\}$ is the graph with $V(C_n) = \{a_1, a_2, a_3, \ldots, a_n\}$ and $E(C_n) = \{a_1a_2, a_2a_3, \ldots, a_{n-1}a_n, a_na_1\}$.

Definition 2.4 A graph $K_n = (V(K_n), E(K_n))$ is called a complete graph of order n when xy is an edge in K_n for every distinct pair $x, y \in V(K_n)$.

Definition 2.5 A complete bipartite graph is a graph whose vertex set can be partitioned into V_1 and V_2 such that every edge joins a vertex in V_1 with a vertex in V_2, and every vertex in V_1 is adjacent to every vertex in V_2.

Definition 2.6 Let G and H be graphs of order m and n, respectively. The corona of two graphs G and H is the graph $G \circ H$ obtained by taking one copy of G and m copies of H, and then joining the ith vertex of G to every vertex of the ith copy of H. The join of vertex v of G and a copy H^v of H in the corona of G and H is denoted by $v + H^v$.

Theorem 2.7 Let G and H be connected non-complete graphs. The subset $S \subset V\,(G \circ H)$ is a fair inverse dominating set of $G \circ H$, if one of the following conditions is satisfied.

(i) $D^v = \{y\}$ is a dominating set of H^v for each $v \in V(G)$ and

a) $S = V(G) \cup (\bigcup_{v \in V(G)} (V(H^v) \setminus D^v))$, or

b) $S = V(G)$, or

c) $S = \bigcup_{v \in V(G)} (V(H^v) \setminus D^v)$

(ii) For each $v \in V(G)$, $S_H^v \subseteq V\,(H^v)$ and

a) $S = \bigcup_{v \in V(G)} S_H^v$, $S_H^v \subset V(H^v)$, and for each $x \in S_H^v$, $xy \in E(H^v)$ for all $y \in V(H^v) \setminus S_H^v$, or

b) $S = \bigcup_{v \in V(G)} V(H^v)$.

***Proof*:** Suppose that statement (i) is satisfied. Then $D^v = \{y\}$ is a dominating set of H^v for each $v \in V(G)$. Clearly, $D = \bigcup_{v \in V(G)} D^v$is a minimum dominating set of $G \circ H$. This means that $V(G \circ H) \setminus D$ is an inverse dominating set with respect to a minimum dominating set D. Consider the following cases.

Case 1. If $S = V(G \circ H) \setminus D$, then $S = V(G) \cup (\bigcup_{v \in V(G)} (V(H^v) \setminus D^v))$ is an inverse dominating set of $G \circ H$ with respect to D. Since $y \in D^v$ for each $v \in V(G)$, $y \notin S$. Thus, $y \in V(G \circ H) \setminus S$. Let $D^v = \{y\}$ and $D^v = \{y'\}$ for all $v, z \in V(G)$. Then,

$$\begin{aligned}
|N_{G \circ H}(y) \cap S| &= |(\{v\} \cup V(H^v) \setminus \{y\}) \cap (V(G) \cup (\bigcup_{v \in V(G)} (V(H^v) \setminus \{y\})))| \\
&= |\{v\} \cup V(H^v) \setminus \{y\}| \\
&= 1 + |V(H^v)| - 1 \\
&= |V(H^v)| \\
&= |V(H^z)| \\
&= 1 + |V(H^z)| - 1 \\
&= |\{z\} \cup V(H^z) \setminus \{y'\}| \\
&= |(\{z\} \cup V(H^z) \setminus \{y'\}) \cap (V(G) \cup (\cup_{v \in V(G)} (V(H^z) \setminus \{y'\})))| \\
&= |N_{G \circ H}(y') \cap S|.
\end{aligned}$$

This implies that every two vertices (y and y') not in Shave the same number of neighbors in S. Hence, S is a fair dominating set of $G \circ H$. Accordingly, S is a fair inverse dominating set of $G \circ H$.

Case 2. If $S \neq V(G \circ H) \setminus D$, then $S \subset V(G \circ H) \setminus D$, that is, $S \subset V(G) \cup (\bigcup_{v \in V(G)} (V(H^v) \setminus D^v))$.
Subcase 1. Consider that $S = V(G)$. Then S is an inverse dominating set of $G \circ H$ with respect to a minimum dominating set $D = \bigcup_{v \in V(G)} D^v$where $D^v = \{y\}$ a dominating set of H^v for each $v \in V(G)$. Let $u, u' \in V(G \circ H) \setminus S$ and$v, v' \in S$ such that $uv, u'v' \in E(G \circ H)$.

$$\begin{aligned} |N_{G \circ H}(u) \cap S| &= |\{v\} \setminus V(G)| \\ &= |\{v\}| \\ &= |\{v'\}| \\ &= |\{v'\} \setminus V(G)| \\ &= |N_{G \circ H}(u') \cap S|. \end{aligned}$$

This implies that every two vertices (u and ') not in S has the same number of neighbors in S. Hence, S is a fair dominating set of $G \circ H$. Accordingly, S is a fair inverse dominating set of $G \circ H$.

Subcase 2. Consider that $S = \bigcup_{v \in V(G)} (V(H^v) \setminus D^v)$. Since $D^v = \{y\}$ is a dominating set of H^v for each $v \in V(G)$, the set $V(H^v) \setminus D^v$ is also a dominating set ofH^v for each $v \in V(G)$. Thus, S is an inverse dominating set of $G \circ H$ with respect to a minimum dominating set $D = \bigcup_{v \in V(G)} D^v$. Let $u, u' \in V(G \circ H) \setminus S$.
If $u, u', v, v' \in V(G)$ such that $uv, u'v' \in E(G)$, then

$$\begin{aligned} |N_{G \circ H}(u) \cap S| &= |(\{v\} \cup (V(H^u) \setminus D^u)) \cap S| \\ &= |V(H^u) \setminus D^u| \\ &= |V(H^{u'}) \setminus D^{u'}| \\ &= |(\{v'\} \cup (V(H^{u'}) \setminus D^{u'})) \cap S| \\ &= |N_{G \circ H}(u') \cap S| \end{aligned}$$

This implies that every two vertices (u and u') not in S has the same number of neighbors in S. Hence, S is a fair dominating set of $G \circ H$. Accordingly, S is a fair inverse dominating set of $G \circ H$.
Similarly, if ($D^v = \{u\}$ and $D^{v'} = \{u'\}$) or ($D^v = \{u\}$ and $u' \in V(G)$), then

$$\begin{aligned} |N_{G \circ H}(u) \cap S| &= |(\{v\} \cup (V(H^u) \setminus D^u)) \cap S| \\ &= |V(H^u) \setminus D^u| \\ &= |V(H^{u'}) \setminus D^{u'}| \\ &= |(\{v'\} \cup (V(H^{u'}) \setminus D^{u'})) \cap S| \\ &= |N_{G \circ H}(u') \cap S|. \end{aligned}$$

that is, S is a fair inverse dominating set of $G \circ H$.

Suppose that statement (ii) is satisfied. Then for each $v \in V(G)$, $S_H^v \subseteq V(H^v)$. Let $D = V(G)$ and consider the following.
Case 1. If $S_H^v \subset V(H^v)$, then $S = \bigcup_{v \in V(G)} S_H^v$, and for each $x \in S_H^v$, $xy \in E(H^v)$ for all $y \in V(H^v) \setminus S_H^v$. Clearly, S_H^v is a dominating set of H^v for each $v \in V(G)$. Thus, $S = \bigcup_{v \in V(G)} S_H^v$, is a dominating set of $G \circ H$. Since $D = V(G)$ is a minimum dominating set of $G \circ H$, $S \subset V(G \circ H) \setminus D$, is an inverse dominating set of $G \circ H$ with respect to a minimum dominating set D. Let $z, w \in V(G \circ H) \setminus S$ and consider the following subcases.

Subcase 1. If z, $w \in V(H^v) \setminus S_H^v$ for each $v \in V(G)$, then $|N_{G \circ H}(z) \cap S| = |N_{H^v}(z) \cap S_H^v| = |S_H^v| = |N_{H^v}(w) \cap S_H^v = N_{G \circ H}(w) \cap S|$. Thus, $|N_{G \circ H}(z) \cap S| = |N_{G \circ H}(w) \cap S|$ for every two distinct vertices z and w from $V(G \circ H) \setminus S$.

Subcase 2. If z, $w \in V(G)$, then $|N_{G \circ H}(z) \cap S| = |N_G(z) \cap S_H^z| = |S_H^z| = |S_H^w| = |N_G(w) \cap S_H^w| = |N_{G \circ H}(w) \cap S|$. Thus, $|N_{G \circ H}(z) \cap S| = |N_{G \circ H}(w) \cap S|$ for every two distinct vertices z and w from $V(G \circ H) \setminus S$.

Subcase 3. If $z \in V(H^v) \setminus S_H^v$ for each $v \in V(G)$ and $w \in V(G)$, then $|N_{G \circ H}(z) \cap S| = |N_{H^v}(z) \cap S_H^v| = |S_H^v| = |S_H^w| = |N_G(w) \cap S_H^w| = |N_{G \circ H}(w) \cap S|$. Thus, $|N_{G \circ H}(z) \cap S| = |N_{G \circ H}(w) \cap S|$ for every two distinct vertices z and w from $V(G \circ H) \setminus S$.

Hence, S is a fair dominating set of $G \circ H$. Accordingly, S is a fair inverse dominating set of $G \circ H$.

Case 2. If $S_H^v = V(H^v)$, then $S = \bigcup_{v \in V(G)} V(H^v) = V(G \circ H) \backslash D$ is an inverse dominating set of $G \circ H$ with respect to a minimum dominating set D. Let $u, v \in V(G \circ H) \setminus S = V(G)$. Then $|N_{G \circ H}(u) \cap S| = |N_{H^u}(u) \cap S_H^u| = |S_H^u| = |S_H^v| = |N_{H^v}(v) \cap S_H^v| = |N_{G \circ H}(v) \cap S|$. Thus, $|N_{G \circ H}(u) \cap S| = |N_{G \circ H}(v) \cap S|$ for every two distinct vertices z and w from $V(G \circ H) \setminus S$. Hence, S is a fair dominating set of $G \circ H$. Accordingly, S is a fair inverse dominating set of $G \circ H$.□

The following result is an immediate consequence of Theorem 2.7.

Corollary 2.8 Let G and H be connected non-complete graphs. Then

$$\gamma_{fd}^{(-1)}(G \circ H) = \begin{cases} |V(G)|, & if\ \gamma(H) = 1, \\ |V(G)| \cdot \gamma_{fd}(H), & if\ otherwise. \end{cases}$$

***Proof*:** Suppose that $\gamma(H) = 1$. Let $D^v = \{y\}$ be a dominating set of H^v for each $v \in V(G)$ and $S = V(G)$. Then S is a fair inverse dominating set of $G \circ H$ by Theorem 2.7. Thus, $\gamma_{fd}^{(-1)}(G \circ H) \leq |S|$. Since the $\gamma(G \circ H) = |V(G)|$, it follows that $|V(G)| = \gamma(G \circ H) \leq \gamma_{fd}^{(-1)}(G \circ H) \leq |V(G)|$. Hence, $\gamma_{fd}^{(-1)}(G \circ H) = |V(G)|$.

Suppose that $\gamma(H) \neq 1$. Let $S = \bigcup_{v \in V(G)} S_H^v$, $S_H^v \subset V(H^v)$, and for each $x \in S_H^v$, $xy \in E(H^v)$ for all $y \in V(H^v) \setminus S_H^v$. Then S is a fair inverse dominating set of $G \circ H$ by Theorem 2.7. Thus,

$$\gamma_{fd}^{(-1)}(G \circ H) \leq |S| = |\bigcup_{v \in V(G)} S_H^v| = \sum_{v \in V(G)} |S_H^v| = |V(G)| \cdot |S_H|,$$

that is, $\gamma_{fd}^{(-1)}(G \circ H) \leq |V(G)| \cdot |S_H|$. Suppose S_H is not a fair dominating set of H. Then there exists u, $u' \in V(H) \setminus S_H$ such that $|N_H(u) \cap S_H| \neq |S_H(u') \cap S_H|$, that is, $|N_{G \circ H}(u) \cap S| \neq |S_{G \circ H}(u') \cap S|$. This contradicts the assumption that S is a fair dominating set of $G \circ H$. Thus, S_H must be a fair dominating set of H. Hence, $\gamma_{fd}^{(-1)}(G \circ H) \leq |V(G)| \cdot |S_H|$ for all fair dominating set S_H of H, that is, $\gamma_{fd}^{(-1)}(G \circ H) = |V(G)| \cdot \gamma_{fd}(H)$.□

3. Conclusion

In this work, we introduced a new parameter of domination in graphs - the fair inverse domination in graphs. The fair inverse domination in the corona of two graphs was characterized. The exact fair inverse domination number resulting from this binary operation of two graphs was computed. This study will pave the way to new research such as bounds and other binary operations of two graphs. Other parameters involving fair inverse domination in graphs may also be explored. Finally, the characterization of fair inverse domination in graphs and its bounds is a promising extension of this study.

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