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Outer-Connected Inverse Domination in the Corona of Two Graphs

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Abstract: Let G be a connected simple graph. A subset S of V(G) is a dominating set of G if for every $v \in V(G) \setminus S$, there exists $x \in S$ such that $xv \in E(G)$. A set $D \subseteq V(G)$ is said to be an outer-connected dominating set in G if D is dominating and either $D = V(G)$ or $\langle V(G) \setminus D \rangle$ is connected. Let D be a minimum dominating set of G. A nonempty subset $S \subseteq V(G) \setminus D$ is an outer-connected inverse dominating set of G if S is an inverse dominating set with respect to D and the subgraph $\langle V(G) \setminus S \rangle$ induced by $V(G) \setminus S$ is connected. The outer connected inverse domination number of G, is denoted by$\tilde{\gamma}_c^{(-1)}(G)$, that is, the minimum cardinality of an outer connected inverse dominating set of G. In this paper, we initiate the study of the concept, and give the outer-connected inverse domination number of some special graphs. Further, we give the characterization and domination number of the outer-connected inverse dominating set in the corona of two nontrivial connected graphs.

Keywords: dominating set, inverse dominating set, outer-connected dominating set, outer-connected inverse dominating set

1. Introduction

Domination in graph was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [1]. Following an article [2] by Ernie Cockayne and Stephen Hedetniemi in 1977, the domination in graphs became an area of study by many researchers. A subset S of $V(G)$ is a *dominating set* of Gif for every $v \in V(G) \setminus S$, there exists $x \in S$ such that $xv \in E(G)$, i.e., $N[S] = V(G)$. The *domination number*$\gamma(G)$ of G is the smallest cardinality of a dominating set of G. Some studies on domination in graphs were found in the papers [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13].

A set S of vertices of a graph G is an outer-connected dominating set if every vertex not in$S$ is adjacent to some vertex in S and the sub-graph induced by$V(G) \setminus S$ is connected. The outer-connected domination number $\tilde{\gamma}_c(G)$ is the minimum cardinality of the outer-connected dominating set S of a graph G. The concept of outer-connected domination in graphs was introduced by Cyman [14]. Some related studies of outer-connected domination in graphs are found in [15, 16, 17, 18, 19, 20, 21].

Let D be a minimum dominating set in G. The dominating set $S \subseteq V(G) \setminus D$is called an *inverse dominating set* with respect to D. The minimum cardinality of an inverse dominating set is called an inverse domination number of G and is denoted by $\gamma^{-1}(G)$. An inverse dominating set of cardinalities $\gamma^{-1}(G)$is called γ^{-1}- set of G. The *inverse domination in a graph* was first found in the paper of Kulli [22] and can be read in the papers [23, 24, 25, 26, 27, 28, 29, 30, 31, 32].

Motivated by the introduction of the outer-connected dominating sets and the inverse dominating sets, a new variant of domination in graphs is introduced in this paper. Let D be a minimum dominating set of G. A nonempty subset $S \subseteq V(G) \setminus D$ is an outer connected inverse dominating set of G, if S is an inverse dominating set with respect to D and the subgraph $\langle V(G) \setminus S \rangle$ induced by $V(G) \setminus S$ is connected. The outer connected inverse domination number of G, is denoted by $\tilde{\gamma}_c^{(-1)}(G)$, that is the minimum cardinality of an outer connected inverse dominating set of G. In this paper, we initiate the study of the concept and give the outer-connected inverse domination number of some special graphs. Further, we show the characterization of the outer-connected inverse dominating set in the join of two nontrivial connected graphs.

For the general terminology in graph theory, readers may refer to [33]. A *graph* G is a pair $(V(G), E(G))$, where $V(G)$ is a finite nonempty set called the *vertex-set* of G and $E(G)$ is a set of unordered pairs $\{u, v\}$ (or simply uv) of distinct elements from $V(G)$ called the *edge-set* of G. The elements of $V(G)$ are called vertices and the cardinality $|V(G)|$of $V(G)$ is the order of G. The elements of $E(G)$ are called edges and the cardinality $|E(G)|$ of $E(G)$ is the size of G. If $|V(G)| = 1$, then G is called a trivial graph. If $E(G) = \emptyset$, then G is called an empty graph. The *open neighborhood* of a vertex $v \in V(G)$ is the set $N_G(v) = \{u \in V(G) : uv \in E(G)\}$. The elements of $N_G(v)$ are called *neighbors* of v. The *closed neighborhood* of $v \in V(G)$is the set $N_G[v] = N_G(v) \cup \{v\}$.If $X \subseteq V(G)$, the *open neighborhood* of X in G is the set $N_G(X) = \bigcup_{v\in X} N_G(v)$. The closed neighborhood of X in G is the set $N_G[X] = \bigcup_{v\in X} N_G[v] = N_G(X) \cup X$. When no confusion arises, $N_G[x]$ [res. $N_G(x)$] will be denoted by $N[x]$ [resp. $N(x)$].

2. Results

Definition 2.1 A simple graphGis an undirected graph with no loop edges or multiple edges.

Definition 2.2 The path$\mathrm{P_n} = \{\mathrm{a_1 a_2 a_3 \dots a_n}\}$ is the graph with $\mathrm{V(P_n)} = \{\mathrm{a_1, a_2, a_3, \dots, a_n}\}$and $\mathrm{E(P_n)} = \{\mathrm{a_1a_2, a_2a_3, \dots, a_{n-1}a_n}\}$.

Definition 2.3 The cycle $\mathrm{C_n} = \{\mathrm{a_1a_2a_3 \dots a_na_1}\}$ is the graph with $\mathrm{V(C_n)} = \{\mathrm{a_1, a_2, a_3, \dots, a_n}\}$ and $\mathrm{E(C_n)} = \{\mathrm{a_1a_2, a_2a_3, \dots, a_na_1}\}$.

Definition 2.4A graph $\mathrm{K_n} = \big(\mathrm{V(K_n), E(K_n)}\big)$ is called a complete graph of order n when xy is an edge in $\mathrm{K_n}$ for every distinct pair $\mathrm{x, y} \in \mathrm{V(K_n)}$.

Definition 2.5 A complete bipartite graph is a graph whose vertex set can be partitioned into $\mathrm{V_1}$ and $\mathrm{V_2}$ such that every edge joins a vertex in $\mathrm{V_1}$ with a vertex in $\mathrm{V_2}$, and every vertex in $\mathrm{V_1}$ is adjacent with every vertex in $\mathrm{V_2}$.

Proposition 2.6 Let $\mathrm{G} = \mathrm{C_n}$. Then $\tilde{\gamma}_c^{(-1)} = \begin{cases} 1, & \text{if } n = 3 \\ 2, & \text{if } n = 4 \\ \text{none}, & \text{if } n \geq 5 \end{cases}$

Proof: Suppose that $G = C_n$. Let $V(C_n) = \{x_1, x_2, \dots, x_n\}$. If $n = 3$, then the set $D = \{x_1\}$ is a minimum dominating set of C_3 and $S = \{x_2\}$ is a minimum inverse dominating set of C_3 with respect to a minimum dominating set D. Since the subgraph induced by $V(C_3) \setminus S = \{x_1, x_2\}$ is connected, it follows that S is a minimum outer-connected inverse dominating set of C_3. Hence, $\tilde{\gamma}_c^{(-1)}(C_3) = |S| = 1$. If $n = 4$, then the set $D = \{x_1, x_2\}$ is a minimum dominating set of C_4 with respect to a minimum dominating set D. Since the subgraph induced by $V(C_4) \setminus S = \{x_1, x_2\}$ is connected, it follows that S is a minimum outer-connected inverse dominating set of C_4. Hence, $\tilde{\gamma}_c^{(-1)}(C_4) = |S| = 2$. If $n \geq 5$, say $n = 5$, then the set $D = \{x_1, x_3\}$ is a minimum dominating set of C_5 and $S = \{x_2, x_4\}$ is a minimum inverse dominating set of C_5 with respect to a minimum dominating set D. Since the subgraph induced by $V(C_5) \setminus S = \{x_1, x_3, x_5\}$ is not connected, it follows that S is not an outer-connected dominating set of C_5. Hence, the outer-connected inverse dominating set in C_5 is none. Similarly, if $n > 5$, then the outer-connected inverse dominating set in C_n is none. ■

Proposition 2.7 Let $G = P_n$. Then $\tilde{\gamma}_c^{(-1)} = \begin{cases} 1, & if\ n = 2 \\ 2, & if\ n = 3\ or\ n = 4 \\ none, & if\ n \geq 5 \end{cases}$

Proof. Suppose that $G = P_n$. Let $V(P_n) = \{x_1, x_2, \dots, x_n\}$. If $n = 2$, then the set $D = \{x_1\}$ is a minimum dominating set of P_2 and $S = \{x_2\}$ is a minimum inverse dominating set of P_2 with respect to a minimum dominating set D. Since the subgraph induced by $V(P_2) \setminus S = \{x_1\}$ is trivially connected, it follows that S is a minimum outer-connected inverse dominating set of P_2. Hence, $\tilde{\gamma}_c^{(-1)}(P_2) = |S| = 1$. If $n = 3$, then the set $D = \{x_2\}$ is a minimum dominating set of P_3 and $S = \{x_1, x_3\}$ is a minimum inverse dominating set of P_3 with respect to a minimum dominating set D. Since the subgraph induced by $V(P_3) \setminus S = \{x_2\}$ is trivially connected, it follows that S is a minimum outer-connected inverse dominating set of P_3. Hence, $\tilde{\gamma}_c^{(-1)}(P_3) = |S| = 2$. If $n = 4$, then the set $D = \{x_2, x_3\}$ is a minimum dominating set of P_4 and $S = \{x_1, x_4\}$ is a minimum inverse dominating set of P_4 with respect to a minimum dominating set D. Since the subgraph induced by $V(P_4) \setminus S = \{x_2, x_3\}$ is connected, it follows that S is an outer-connected dominating set of P4. Hence, $\tilde{\gamma}_c^{(-1)}(P_4) = |S| = 2$. If $n = 5$, then the set $D = \{x_2, x_5\}$ is a minimum dominating set of P_5 and $S = \{x_1, x_4\}$ is a minimum inverse dominating set of P_5 with respect to a minimum dominating set D. Since the subgraph induced by $V(P_5) \setminus S = \{x_2, x_3, x_5\}$ is not connected, it follows that S is not an outer-connected dominating set of P_5 .Hence, the outer-connected inverse dominating set in P_5 is none. Similarly, if $n > 5$, then the outer-connected inverse dominating set in P_n is none. ■

Remark 2.8 Let G be a special graph.

i. if $G = K_n$, then $\tilde{\gamma}_c^{(-1)}(G) = 1, \ \forall n \geq 2$
ii. if $G = S_n$, then $\tilde{\gamma}_c^{(-1)}(G) = n, \ \ \forall n \geq 1$
iii. if $G = K_{m,n}$, then $\tilde{\gamma}_c^{(-1)}(G) = 2, \ \ \forall m, n \geq 2$

Definition 2.9 Let G and H be graphs of order m and n, respectively. The corona of two graphs G and H is the graph $G \circ H$ obtained by taking one copy of $G$and m copies of H, and then joining the $it\,h$ vertex of G to every vertex of the $it\,h$ copy of H. The join of vertex v of G and a copy H^v of H in the corona of G and H is denoted by $v + H^v$.

The following result give the characterization of an outer-connected inverse domination in the corona of two graphs.

Theorem 2.10 Let$G$ and H be nontrivial connected graphs. The subset $S \subset V(G \circ H)$ is an outer-connected inverse dominating set of $G \circ H$, if one of the following conditions is satisfied.

i. $S = \left(\cup_{v \in V(G)} (V(H^v) \setminus D^v)\right)$, where $D^v = \{y\}$ is a dominating set of H^v for each $v \in V(G)$.
ii. $S = (\cup_{v \in V(G)} S_H^v)$, where $S_H^v \subset V(H^v) \setminus D^v$ is a dominating set of H^v, $D^v = \{y\}$ is a dominating set of H^v for each $v \in V(G)$.
iii. $S = \cup_{v \in V(G)} S_H^v$, where S_H^v is a dominating set of H^v for each $v \in V(G)$.

Proof. Suppose that statement (i) is satisfied. Then $S = \cup_{v \in V(G)} (V(H^v) \setminus D^v)$, where $D^v = \{y\}$ is a dominating set of H^v for each $v \in V(G)$. This implies that $V(H^v) \setminus D^v$ is a dominating set of H^v for each $v \in V(G)$. Clearly, $D = \cup_{v \in V(G)} D^v$ is a minimum dominating set of $G \circ H$ and $S = \cup_{v \in V(G)} (V(H^v) \setminus D^v)$ is a dominating set of $G \circ H$. Thus, $V(G \circ H) \setminus D = V(G) \cup \left(\cup_{v \in V(G)} (V(H^v) \setminus D^v)\right)$ is an inverse dominating set of $G \circ H$ with respect to a minimum dominating set D. Since $S = \cup_{v \in V(G)} (V(H^v) \setminus D^v) \subset V(G \circ H) \setminus D$, it follows that S is an inverse dominating set of $G \circ H$ with respect to a minimum dominating set D. Since $S = \cup_{v \in V(G)} (V(H^v) \setminus D^v)$, it follows that for each $u \notin S$, there exists $v \in V(G) \nsubseteq S$ such that $uv \in E(G \circ H)$. Thus, the subgraph induced by $V(G \circ H) \setminus S$ is connected. Hence S is an outer-connected dominating set of $G \circ H$. Accordingly, S is an outer-connected inverse dominating set of $G \circ H$.

Suppose that statement (ii) is satisfied. Then $S = \cup_{v \in V(G)} S_H^v$, where $S_H^v \subset V(H^v) \setminus D^v$ is a dominating set of H^v , $D^v = \{y\}$ is a dominating set of H^v for each $v \in V(G)$. By using similar arguments in the proof of statement (i), $D = \cup_{v \in V(G)} D^v$ is a minimum dominating set of $G \circ H$ and$S = \cup_{v \in V(G)} S_H^v$ is a dominating set of $G \circ H$.Thus, $V(G \circ H) \setminus D = V(G) \cup \left(\cup_{v \in V(G)} (V(H^v) \setminus D^v)\right)$is an inverse dominating set of $G \circ H$ with respect to a minimum dominating set D. Since$S = \cup_{v \in V(G)} S_H^v \subset (V(H^v) \setminus D^v)$ is a dominating set of $G \circ H$, it follows that S is an inverse dominating set of $G \circ H$ with respect to a minimum dominating set D. Since$S = \cup_{v \in V(G)} S_H^v$, it follows that for each $u \notin S$, there exists $v \in V(G) \nsubseteq S$ such that $uv \in E(G \circ H)$. Thus, the subgraph induced by $V(G \circ H) \setminus S$ is connected. Hence S is an outer-connected dominating set of $G \circ H$. Accordingly, S is an outer-connected inverse dominating set of $G \circ H$.

Suppose that statement (iii) is satisfied. $S = \cup_{v \in V(G)} S_H^v$, where S_H^v is a dominating set of H^v for each $v \in V(G)$. Let $D = V(G)$. Then D is a minimum dominating set of $G \circ H$ and $V(G \circ H) \setminus D = \cup_{v \in V(G)} \ V(H^v)$is an inverse dominating set of $G \circ H$with respect to a minimum dominating set D. Since $S = \cup_{v \in V(G)} S_H^v \subseteq \cup_{v \in V(G)} V(H^v)$, it follows that S is an inverse dominating set of $G \circ H$ with respect to a minimum dominating set D. If $S_H^v = V(H^v)$ for each $v \in V(G)$, then $S = \cup_{v \in V(G)} V(H^v)$. Since for each$v \in V(G) \notin S$, there exists distinct$v' \in V(G)$ such that $uv' \in E(G \circ H)$. Thus, the subgraph induced by $V(G \circ H) \setminus S$ is connected. Hence S is an outer-connected dominating set of $G \circ H$. Accordingly, S is an outer-connected inverse dominating set of $G \circ H$. Similarly, if$S_H^v \subset V(H^v)$ where S_H^v is a dominating set for each $v \in V(H)$, then S is an outer-connected inverse dominating set of $G \circ H$.■

The following result is an immediate consequence of Theorem 2.10.

Corollary 2.11 Let G and H be nontrivial connected graphs. Then $\tilde{\gamma}_c^{(-1)}(G \circ H) = |V(G)| \cdot \gamma(H)$.

Proof. Suppose that $S = \cup_{v\in V(G)} S_H^v$, where S_H^v is a dominating set of H^v for each $v \in V(G)$. Then by Theorem 2.10, S is an outer-connected inverse dominating set of $G \circ H$. This implies that

$$\tilde{\gamma}_c^{(-1)}(G \circ H) \leq |S| = \left|\cup_{v\in V(G)} S_H^v\right| = \Sigma_{v\in V(G)}|S_H^v| = |V(G)| \cdot |S_H|,$$

that is, $\tilde{\gamma}_c^{(-1)}(G \circ H) \leq |V(G)| \cdot |S_H|$, for all dominating set S_H of H. Since $\gamma(H) \leq |S_H|$, it follows that

$$|V(G)| \cdot \gamma(H) \leq |V(G)| \cdot |S_H| \text{ for all dominating set } S_H \text{ of } H.$$

that is, $\tilde{\gamma}_c^{(-1)}(G \circ H) = |V(G)| \cdot \gamma(H) \leq |V(G)| \cdot |S_H|$. ■

3. Conclusion

In this work, we introduced a new parameter of domination in graphs - the outer-connected inverse domination in graphs. The outer-connected inverse domination in the corona of two graphs were characterized. The exact outer-connected inverse domination number resulting from this binary operation of two graphs were computed. This study will pave a way to new research such bounds and other binary operations of two graphs. Other parameters involving outer-connected inverse domination in graphs may also be explored. Finally, the characterization of an outer-connected inverse domination in graphs and its bounds is a promising extension of this study.

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