

## Use of data envelopment analysis to measure the relative efficiency of the collages Sudan university

Ashraf Hassan Idris Brama

*Department of management information system & product management, Collage of Business & Economics, Qassim university, Buraydah, Kingdom of Saudi Arabia*

**Abstract:** In a relatively short period of time Data Envelopment Analysis (DEA) has grown into a powerful quantitative, analytical tool for measuring and evaluating performance. DEA has been successfully applied to a host of different types of collages of university the objective to use one of the modern quantitative methods, to gauge the relative internal efficiency to collages of Sudan University, the results of the study as the college of engineering and oil technology, achieved complete relative efficiency in both the (BCC, CCR), recommends the Data base is essential in the university and it should be inclusive of all educational variables relevant to the university, periodically updated and accessible to researchers. Because the main problem facing those who want to implement this method is the unavailability of data and inclusion.

**Keywords:** Data envelopment analysis (DEA), academic faculties, relative efficiency, optimization

### 1- Introduction

Data Envelopment Analysis (DEA) is a relatively new “data oriented” approach for evaluating the performance of a set of peer entities called Decision Making Units (DMUs) which convert multiple inputs into multiple outputs. The definition of a DMU is generic and flexible. Recent years have seen a great variety of applications of DEA for use in evaluating the performances of many different kinds of entities engaged in many different activities in many different contexts in many different countries. These DEA applications have used DMUs of various forms to evaluate the performance of entities, such as hospitals, US Air Force wings, universities, cities, courts, business firms, and others, including the performance of countries, regions, etc. Because it requires very few assumptions, DEA has also opened up possibilities for use in cases which have been resistant to other approaches because of the complex (often unknown) nature of the relations between the multiple inputs and multiple outputs involved in DMUs.

Since DEA in its present form was first introduced in 1978, researchers in a number of fields have quickly recognized that it is an excellent and easily used methodology for modeling operational processes for performance evaluations. This has been accompanied by other developments. For instance, Zhu (2002) provides a number of DEA spreadsheet models that can be used in performance evaluation and benchmarking. DEA’s empirical orientation and the absence of a need for the numerous a priori assumptions that accompany other approaches (such as standard forms of statistical regression analysis) have resulted in its use in a number of studies involving efficient frontier estimation in the governmental and nonprofit sector, in the regulated sector, and in the private sector. See, for instance, the use of DEA to guide removal of the Diet and other government agencies from Tokyo to locate a new capital in Japan, as described in Takamura and Tone (2003).

Since the introduction of the CCR model by professors Abraham Charnes, William Cooper and Edwardo Rhodes in 1978, the way scholars investigate the efficiency and productivity of organizations shifted drastically. The so-called Data Envelopment Analysis (DEA) was different from statistical procedures comparing measures of performance based on an average observation. Based on Farrell’s seminal work on the measurement of productive efficiency, the problem of measuring the technical efficiency of Decision Making Units (DMUs) became a matter of how far production is expanded without using additional resources. Measuring the technical efficiency is made by comparing the DMU performance with a hypothetical unit constructed as a weighted average of other observed firms. The interpretation behind Farrell concepts is that if a decision unit can transform input resources into output production in a Pareto-efficient way (i.e. in such a way that there is no other configuration with more production at the same level of resources, or fewer resources resulting in the same level of production) then another unit with similar scale must be capable of producing similar results.

### 2- DEA methodology

DEA is a powerful non-parametric method in efficiency evaluation. It is widely used in various sectors recently (Alper et al. 2015; LaPlante and Paradi 2015; Misiunas et al. 2015; Zografidou et al. 2015) as well as academia. DEA gives an efficiency score by dealing with multiple inputs and multiple outputs. The relative

efficiency of a DMU is calculated relative to all other DMUs (McMillan and Chan, 2006). In our case study, 12 faculties of a leading Turkish university specify the DMUs. The traditional DEA approach is proposed by Charnes et al. (1978). It is known as CCR model and calculates pure technical efficiency and scale efficiency scores. Then another version of DEA is developed by Banker et al. (1984). It is known as BCC model and calculates only pure technical efficiency scores. The main difference between BCC and CCR models is the treatment of returns to scale. The CCR model is based on the evaluation of constant returns to scale (CRS). The BCC model is on variable returns to scale (VRS) (Abbott and Doucouliagos, 2003; Colbert et al. 2000). For each model, fractional programming linear programming transformation is used considering input-oriented and output oriented forms. Input-oriented DEA models express the reductions it would be required to make in the inputs of the assessed DMU so that it can be become qualified as efficient. Similarly, output-oriented DEA models identify the necessary increase in output to achieve the same effect.

The Data Envelopment Analysis (DEA) methodology introduced by Abraham Charnes and colleagues estimates an efficiency frontier by considering the best performance observations (extreme points) which “envelop” the remaining observations using mathematical programming techniques. The concept of efficiency can be defined as a ratio of produced outputs to the used inputs

$$Efficiency = \frac{outputs}{inputs}$$

So that an inefficient unit can become efficient by expanding products (output) keeping the same level of used resources, or by reducing the used resources keep the same production level, or by a combination of both. Considering  $j = 1, 2, 3, \dots, m$  Decision Making Units (DMUs) using  $x_i \mid i = 1, 2, 3, \dots, n$  inputs to produce  $y_r \mid r = 1, 2, 3, \dots, s$  outputs and prices (multipliers)  $v_i$  and  $u_r$  associated with those inputs and outputs, we can also formalize the efficiency expression in (1) as the ratio of weighted outputs to weighted inputs:

$$Efficiency = \frac{\sum_{r=1}^s u_r y_{jr}}{\sum_{i=1}^n v_i x_{ji}}$$

In Charnes et al. (1978) DEA methodology the multipliers, and a measure for the technical efficiency for a specific DMU can be estimated by solving the fractional programming problem:

$$\max \frac{\sum_{r=1}^s u_r y_{or}}{\sum_{i=1}^n v_i x_{oi}} \mid \sum_{r=1}^s u_r y_{jr} - \sum_{i=1}^n v_i x_{ji} \leq 0$$

### 3- Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) was introduced by Charnes, Cooper, Dan Rhodes. DEA method was made as tool to evaluate performance an activity in entity units or organization. DEA was nonparametric approach which is basically a technique linear programming. The researchers on several fields quickly admitted that DEA was excellent methodology to modelling operational process. On DEA, Companies were called as decision making units (DMUs). Term decision making units (DMUs) was introduced in the same way as entities, each of entities be evaluated as part of group that utilize inputs to producing outputs. The result of evaluation made in score efficiency about between 0 until 1 and represent degree of efficiency which is obtained from entities assessed. With the score, DEA also identified source and number of not efficient DMUs on every input and output. It also identified DMU (that placed on efficient frontier) that included actively in the appearance of these results. Assessment entities were all efficient DMUs and hence can function as benchmarking to effective improvement at future performance from DMUs be evaluated [4]. Some basic DEA models and expansion DEA models used in this paper that is: CCR model, BCC model, and Slack-Based (SBM) model

#### 3-1 CCR Model

To allow for applications to a wide variety of activities, we use the term Decision Making Unit (=DMU) to refer to any entity that is to be evaluated in terms of its abilities to convert inputs into outputs. These evaluations can involve governmental agencies and not-for-profit organizations as well as business firms. The evaluation can also be directed to educational institutions and hospitals as well as police forces (or subdivision thereof) or army units for which comparative evaluations of their performance are to be made.

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We assume that there are  $n$  DMUs to be evaluated. Each DMU consumes varying amounts of  $M$  different inputs to produce  $s$  different outputs. Specifically,  $DMU_j$  consumes amount  $x_{ij}$  of input  $i$  and produces amount  $y_{rj}$  of output  $r$ . We assume that  $x_{ij} \geq 0$  and  $y_{rj} \geq 0$  and further assume that each DMU has at least one positive input and one positive output value.

We now turn to the “ratio-form” of DEA. In this form, as introduced by Charnes, Cooper, and Rhodes, the ratio of outputs to inputs is used to measure the relative efficiency of the  $DMU_j = DMU_0$  to be evaluated relative to the ratios of all of the  $j = 1, 2, \dots, n$ . We can interpret the CCR construction as the reduction of the multiple-output /multiple-input situation (for each DMU) to that of a single ‘virtual’ output and ‘virtual’ input. For a particular DMU the ratio of this single virtual output to single virtual input provides a measure of efficiency that is a function of the multipliers. In mathematical programming parlance, this ratio, which is to be maximized, forms the objective function for the particular DMU being evaluated, so that symbolically

$$\begin{aligned} e_0 &= \max \sum_r u_r y_{r0} \\ \text{s.t. } \sum u_r y_{r0} - \sum u_i x_{i0} &\leq 0 \\ \sum u_i x_{i0} &= 1 \\ u_r, u_i &\geq 0 \end{aligned}$$

The model is run  $n$  times in identifying the relative efficiency scores of all the DMUs. Each DMU selects a set of input weights  $v_i$  and output weights  $u_r$  that maximize its efficiency score. Generally, a DMU is efficient if it obtains the maximum score of 1, else a DMU is inefficient.

The DEA model that implemented Variable Return to Scale (VRS) is called BCC model. In BCC model, VRS is assumed and the efficient frontier is formed by the convex hull of the existing DMUs.

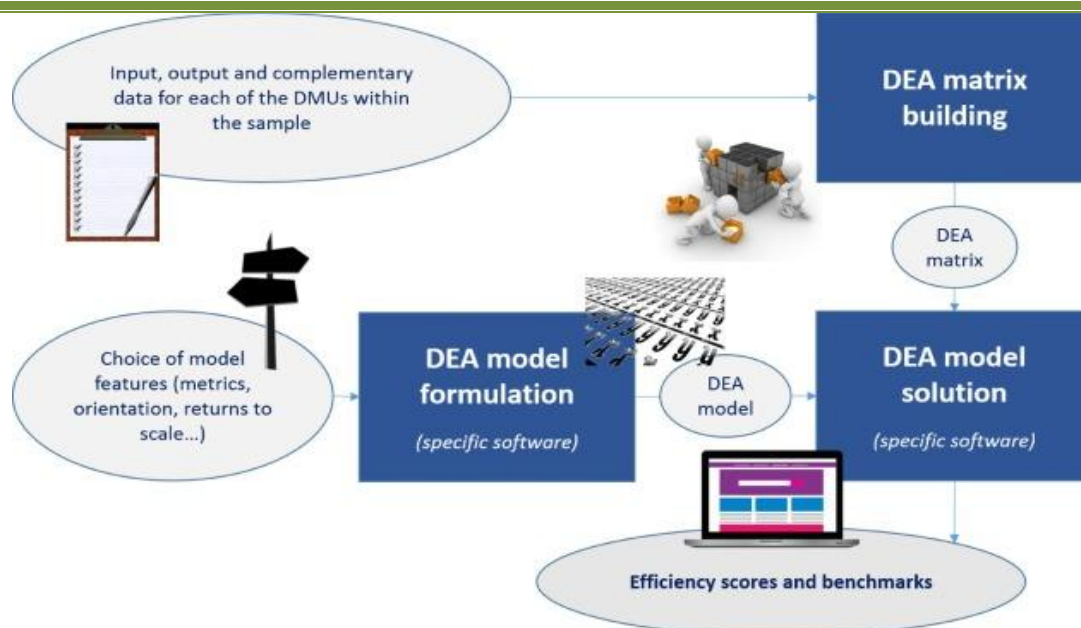
### 3-2 BCC Model

DEA model that implemented Variable Return to Scale (VRS) was known as model BCC, Banker, Charnes dan Cooper (1984). In BCC model, VRS is assumed and the efficient frontier is formed by the convex hull of the existing DMUs. The envelopment form of BCC is:

$$\begin{aligned} \text{Min } \theta_0 \\ \text{s.t.} \\ \sum \lambda_r x_{ij} - \theta_0 x_{i0} &\leq 0 \\ \sum \lambda_r y_{rj} - y_{r0} &\geq 0 \\ \sum \lambda_i &= 1 \\ \lambda_i &\geq 0 \end{aligned}$$

### 4- Data envelopment analysis

DEA is a linear programming methodology that empirically quantifies the relative efficiency of multiple similar entities or DMUs (Cooper et al., 2007). The DMU is the homogeneous entity responsible for the conversion of inputs into outputs. As shown in Fig. 16.3, to carry out a DEA study, a matrix composed of the inputs, outputs, and complementary elements of the sample of DMUs is required. Once the DEA model has been formulated according to a set of features such as metrics and orientation, the matrix is implemented in the model to be solved, thus obtaining as main results relative efficiency scores and operational benchmarks for each DMU.



The relative efficiency scores are calculated through a nonparametric procedure based only on the observed data and basic assumptions for the resolution of an optimization model. For each DMU, an efficiency score ( $\phi$ ) is obtained. In addition, for the DMUs identified as inefficient (i.e.,  $\phi < 1$ ), a set of target values.

### 5- Establishment of the model

In this study, we deal with the efficiency evaluation of the faculties of the observed university using DEA methodology. DEA Solver is used to measure the technical efficiency of the faculties based on both CCR and BCC input oriented models. As mentioned earlier, most of previous studies only used BCC model. In this study, both CCR and BCC input oriented models are used to select the model that fairly represents the behavior of the system. Due to the fact that in a university environment, it is easier to control the inputs rather than the outputs, the DEA input oriented model is preferred to compute the efficiency of these faculties.

### 6- Data analysis:

Data Envelopment Analysis Results The results obtained from the output-oriented DEA – BCC model with MAXDEA 8 PRO software to estimate the efficiency of in 9 collage of university, and obtains the technical efficiency, pure technical efficiency and scale efficiency.

Table (1) illustrate descriptive statistic of data

Variable	Mean	Maximum value	Minimum value	Standard deviation
Staff	68	206	7	15.03
Student	1365	8819	150	25.10
Credit hours	160	160	154	10.25
Graduate	480	700	80	70.85

Table (2) illustrate the model CCR and BBC

Collage	CCR-O	BBC-O
science	0.72	0.83
medicine	0.89	0.91
Medical Laboratory	0.85	0.88
Oil technology	1	1
education	0.99	0.88
Engineering	1	1
Veterinary Medicine	0.85	0.87
Radiology Sciences	0.84	0.89
Music and Drama	0.93	0.79

Source: DEA-Solver 2023

Table (3) illustrate optimization level by Relative Efficiency Models for collage science

Income and outcome	CCR-O				College Reference	BBC-O			College Reference
	actual value	target value	optimization	Percent %		target value	optimization	Percent %	
Staff	206	197.78	-8.22	-0.039	Oil technology	19.03	-10.96	-36.54	Oil technology and Engineering
Student	2635	2635	0.00	0.0		131	0.0	0.0	
Credit hours	1280	1280	0.00	0.0		1016.54	17.46	-1.69	
Graduate	89	122.95	33.95	38.15		120.30	31.30	35.18	

Source: DEA-Solver 2023

From table (3) illustrate that the collage science did not relative efficiency has not been achieved to (CCR) for relative efficiency must to be decrease the staff to (8) by (0.039%) and increase graduate to (31.30) by (35.18) for efficiency achieved by college Oil technology and Engineering.

Table (4) illustrate optimization level by Relative Efficiency Models for collage medicine

Income and outcome	CCR-O				College Reference	BBC-O			College Reference
	actual value	target value	optimization	Percent %		target value	optimization	Percent %	
Staff	50	36	-24	--0.48	Oil technology	16.21	33.78	-67.58	Oil technology and Engineering
Student	102	102	0.0	0.0		102	0.0	0.0	
Credit hours	800	800	0.0	0.0		800	0.0	0.0	
Graduate	86	95.65	9.65	11.23		95.01	9.01	10.49	

Source: DEA-Solver 2023

From table (4) illustrate that the collage medicine did not relative efficiency has not been achieved to (CCR) for relative efficiency must to be decrease the staff to (24) by (48.0%) and increase graduate to (9) by (10.49) for efficiency achieved by college Oil technology and Engineering.

Table (5) illustrate optimization level by Relative Efficiency Models for collage Medical Laboratory

Income and outcome	CCR-O				College Reference	BBC-O			College Reference
	actual value	target value	optimization	Percent %		target value	optimization	Percent %	
Staff	16	14.55	-1.44	-9.01	Oil technology	14.9	-1.09	-6.83	Oil technology and Engineering
Student	95	95	0.0	0.0		95	0.0	0.0	
Credit hours	697	679	0.0	0.0		679	0.0	0.0	
Graduate	75	88.108	13.108	17.48		87.14	21.14	16.2	

Source: DEA-Solver 2023

From table (5) illustrate that the collage Medical Laboratory did not relative efficiency has not been achieved to (CCR) for relative efficiency must to be decrease the staff to (1) by (9.01%) and increase graduate to (21.14) by (16.2) for efficiency achieved by college Oil technology and Engineering.

Table (6) illustrate optimization level by Relative Efficiency Models for collage Oil technology

Income and outcome	CCR-O				College Reference	BBC-O			College Reference
	actual value	target value	optimization	Percent %		target value	optimization	Percent %	
Staff	40	40	0.0	0.0	-	40	0.0	0.0	-
Student	600	600	0.0	0.0		600	0.0	0.0	
Credit hours	750	750	0.0	0.0		750	0.0	0.0	
Graduate	80	80	0.0	0.0		80	0.0	0.0	

Source: DEA-Solver 2023

From table (6) illustrate that the collage Oil technology relative efficiency has been achieved to (CCR) and also relative efficiency has been achieved to (BBC).

Table (7) illustrate optimization level by Relative Efficiency Models for collage of education

Income and outcome	CCR-O				College Reference	BBC-O			College Reference
	actual value	target value	optimization	Percent %		target value	optimization	Percent %	
Staff	106	106	0.0	0.0	Engineering	25	0.0	0.0	-
Student	2309	2309	0.0	0.0		224	0.0	0.0	
Credit hours	1410	1063.22	-346.77	-24.59		1410	0.0	0.0	
Graduate	199	199.73	0.73	0.37		199	0.0	0.0	

Source: DEA-Solver 2023

From table (7) illustrate that the college education did not relative efficiency has not been achieved to (CCR) for relative efficiency must to be decrease the Credit hours to (347) by (25%) and increase graduate to (0.73) by (0.0036) for efficiency achieved by college Oil technology and Engineering.

Table (8) illustrate optimization level by Relative Efficiency Models for collage Engineering

Income and outcome	CCR-O				College Reference	BBC-O			College Reference
	actual value	target value	optimization	Percent %		target value	optimization	Percent %	
Staff	202	202	0.0	0.0	-	202	0.0	0.0	-
Student	8819	8819	0.0	0.0		8819	0.0	0.0	
Credit hours	1600	1600	0.0	0.0		1600	0.0	0.0	
Graduate	1200	1200	0.0	0.0		1200	0.0	0.0	

Source: DEA-Solver 2023

From table (8) illustrate that the collage Engineering technology relative efficiency has been achieved to (CCR) and also relative efficiency has been achieved to (BBC)

Table (9) illustrate optimization level by Relative Efficiency Models for college of Veterinary Medicine

Income and outcome	CCR-O				College Reference	BBC-O			College Reference
	actual value	target value	optimization	Percent %		target value	optimization	Percent %	
Staff	90	89.72	-0.064	-0.28	Oil technology	86.9	-2.82	-12.30	Oil technology and Engineering
Student	422	422	0.0	0.0		422	0.0	0.0	
Credit hours	956	956	0.0	0.0		956	0.0	0.0	
Graduate	165	192	27.39	16.6		190	25	15	

Source: DEA-Solver 2023

From table (9) illustrate that the collage Veterinary Medicine did not relative efficiency has not been achieved to (CCR) for relative efficiency must to be decrease the staff to (0.064) by (0.28%) and increase graduate to (27.39) by (16.6) for efficiency achieved by college Oil technology and Engineering.

Table (10) illustrate optimization level by Relative Efficiency Models for College of Radiology Sciences

Income and outcome	CCR-O				College Reference	BBC-O			College Reference
	actual value	target value	optimization	Percent %		target value	optimization	Percent %	
Staff	23	23	0.0	0.0	Engineering	23	0.0	0.0	Oil technology and Engineering
Student	483	483	0.0	0.0		483	0.0	0.0	
Credit hours	739	739.87	-49.12	-6.23		736.81	-52.189	-6.61	
Graduate	61	72	11	18		72	11	17	



Source: DEA-Solver 2023

From table (10) illustrate that the collage Radiology Sciences did not relative efficiency has not been achieved to (CCR) for relative efficiency must to be decrease the Credit hours to (49.12) by (6.23%) and increase graduate to (11) by (18.0%) for efficiency achieved by college Oil technology and Engineering.

Table (11) illustrate optimization level by Relative Efficiency Models for College of Music and Drama

Income and outcome	CCR-O				College Reference	BBC-O			
	actual value	target value	optimization	Percent %		target value	optimization	Percent %	College Reference
Staff	34	30.79	-3.03	-23.37	Engineering	34	0.0	0.0	
Student	413	143	0.0	0.0		413	0.0	0.0	
Credit hours	333	333	0.0	0.0		333	0.0	0.0	
Graduate	114	122.36	8.36	7.33		114	0.0	0.0	

Source: DEA-Solver 2023

From table (11) illustrate that the collage Music and Drama did not relative efficiency has not been achieved to (CCR) for relative efficiency must to be decrease the staff to (3.03) by (23.37%) and increase graduate to (8.36) by (7.3) for efficiency achieved by college Oil technology and Engineering.

**7- Result:**

- 1- Collage engineering and oil technology they achieved relative efficiency for the model (CCR & BCC).
- 2- Other collage didn't achieve relative efficiency for the model (CCR & BCC) except College of Music and Drama and collage of education.
- 3- Results also indicate that efficiency scores from robust data envelopment analysis provide better accuracy.
- 4- Application of data envelopment analysis (RDEA) is appropriate for measuring the efficiency of collages university in organizing high education.

**8- Recommendation:**

Through an application Data Envelopment Analysis to measure relative efficiency for collages Sudan university of science and technology

- 1- This study recommends comparing the Sudan Universities among themselves, or compare the efficiency of similar colleges (e.g. the engineering College) in Sudan Universities with each other.
- 2- It also recommends comparing the Sudan Universities with Armband Foreign Universities to determine the efficiency of using the available resources to them
- 3- further studies using the Data Envelopment Analysis method take into consideration the time horizon to find efficiency indicators at the college level over certain period of time
- 4- The technique should be widely applicable in different other fields

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